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Abstract:

A common form of credit market intervention in many economies is a directed lending policy wherein regulators mandate lenders to supply at least β proportion of their loanable funds to certain underserved sectors. Here we analyse welfare implications of such a policy in the context of a two-sector credit market consisting of underserved and mainstream sectors. Our theoretical results indicate that in the absence of directed lending policy, the optimal loan contract offered to the underserved borrower is unfavourable compared to that of the mainstream borrower, even when both borrowers have an identical project. We deduce that when β lies in (0,0.5), directed lending policy not only mitigates credit rationing of the underserved sector, but also improves total welfare of the economy; within this parameter space, it is possible to derive an optimal level of β that maximizes total welfare.

Key words: credit market intervention, directed lending policy, credit rationing, mandated lending, credit quota policy
JEL codes: G21, G28, D21

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Welfare Analysis of Directed Lending Policy

1. INTRODUCTION

Directed lending policies are a type of credit market intervention wherein regulators direct banks to lend to certain sectors or segments of the economy that are otherwise deemed credit constrained. In the United States of America (USA), the Community Reinvestment Act (CRA, 1977) requires federally insured banks to supply credit to low and moderate income (LMI) households and minority communities in the area of charter of the banks; non-compliance of CRA obligations sometimes leads to denial of regulatory approval for banks' business expansion plans. Similarly, India's Priority Sector Lending Policy (PSLP) mandates commercial banks to advance certain proportion of their loanable funds to certain "priority" sectors such as agriculture, small firms, persons from poor and/or underprivileged caste background etc., and a penalty is imposed on banks that fall short of this quota. Such policies are in place in other countries as well, eg., Philippines (Agri-Agra and Magna Carta Laws) and Brazil (the earmarked credit policy). The most prevalent form of the directed lending policy is the "credit quota" policy wherein a specific proportion of banks' loanable funds are directed to the targeted sector.

The genesis of credit market intervention policies lies in regulators' attempt to address unfavourable allocation of credit to certain underserved but important sectors of an economy. Stiglitz (1992) noted that such intervention policies have a 'social' objective, rather than a narrow 'economic' objective; as they enhance equality of opportunity, whose benefit accrue to the entire economy in the long run. Notwithstanding their social objective, such intervention policies has been debated intensely. One line of argument is that such policies distort market-based outcomes and interfere with lenders' business strategies and hence are economically sub-optimal. Others argue that such policies are desirable from the point of view of equitable allocation of scarce capital, given that credit markets are laden with the problem of credit rationing that may result in underinvestment in good opportunities. Intervention generally takes various forms, such as loan guarantee, interest rate subsidy (also known as credit subsidy), direct government lending, government equity investment in borrowers' project, directed lending policy and so on. Among these policies, the directed lending policy is unique since unlike other forms of interventions, the directed lending policies do not require government funds for their implementation; thus, the welfare consideration of this policy does not involve government's budget and fiscal constraints. Despite this uniqueness, the theoretical literature on the effectiveness of credit market intervention has not covered directed lending policy, even though there is a plethora of theoretical analyses of other forms of intervention. This paper contributes by developing a theoretical model to investigate whether and when directed lending policy as a credit market intervention is welfare improving. We develop a simple model of credit quota-based directed lending policy in the context of credit rationing of borrowers belonging to marginalized/LMI/underserved[1] sectors and analyze the features of the optimal loan contracts with and without directed lending policy. Our theoretical model is based on features of marginalized/LMI/underserved sectors where borrowers are often found to be the credit rationed. We show that in the absence of any intervention, among borrowers having identical projects, lenders will ration out the LMI borrowers. Directed lending policy can improve flow of funds to the LMI sector under certain parametric specifications on the credit quota parameter. Another implication of our analysis is that lenders can still make positive profit, even in the presence of such intervention, contrary to the argument that intervention policies hamper lender's profitability. Finally we show that the regulator's welfare function is concave in the credit-quota parameter implying that it is possible to derive an optimal credit quota that maximizes social welfare.

Section 2 of the paper presents a review of the related literature. In section 3, we present our theoretical model of directed lending policy and provide some analysis of the theoretical results considering situations without and with the intervention through directed lending policy. Thereafter, in Section 4, we present a welfare analysis of directed lending policy followed by conclusion in Section 5.

2. REVIEW OF RELATED LITERATURE

We begin with a brief review of the credit rationing literature. Credit rationing is a phenomenon wherein demand for credit exceeds supply but lenders, instead of raising interest rate to a market clearing level, choose to supply credit to some borrowers while denying to others (fully or partially). Thus, many borrowers are rationed out of the market, even though they may be willing to pay a higher interest rate and even though they may have identical projects as those who receive credit. The literature on credit rationing has interpreted such

[1] In this paper, we use the terms marginalized, LMI (Low and Moderate Income) and underserved interchangeably.

lending behavior as consistent with a profit maximizing lender's rational behavior (Hodgman, 1960; Freimer and Gordon, 1965; Jaffe and Modigliani, 1969; Stiglitz and Weiss, 1981, among others). According to Hodgman (1960), interest rate and the loan amount are systematically interlinked such that an increase in interest rate essentially implies higher volume of repayment for the borrower thus increasing borrower's credit risk when interest rates are increased beyond certain limits. Therefore, the lender would avoid raising interest rates to the market clearing level and hence in the equilibrium there is excess demand for loans. Extending this line of analysis, Freimer and Gordon (1965) classified two types of credit rationing, viz., weak credit rationing in which lender may vary both loan amount and interest rate upto a limit beyond which he refuses to lend (Hodgman type credit rationing), and strict credit rationing, in which the lender sets an interest rate and lends at that interest rate upto a pre-determined volume of loans and refuses to lend more thereafter. Freimer and Gordon (1965) demonstrated that a profit maximizing bank would indulge only in strict credit rationing as according to them, loan amount is interest inelastic and therefore the bank would fix the interest rate and ration credit at that rate. Jaffe and Modigliani (1969) derived that rational lenders' optimal loan offer curve may be "backward bending" function of interest rate and depending on which part of the offer curve intersects a borrower's demand curve, credit rationing may arise endogenously. The explanation by Stiglitz and Weiss (1981) is based on the existence of imperfect and asymmetric information in the credit market due to which lenders cannot observe the riskiness of different projects (ex-ante information asymmetry) and/or the effort level of the borrowers (moral hazard). Raising interest rates to clear the markets, Stiglitz and Weiss (1981) argue, would drive safe borrowers out of the credit market (adverse selection) while inducing risky borrowers to indulge in riskier projects to fetch higher returns (adverse incentive); hence the lender would avoid raising interest rates to market clearing level. On the other hand, the model of Williamson (1986, 1987) assumed that borrowers may not have an incentive to truthfully reveal the outcome of the project. In Williamson's (1986, 1987) costly state verification model, an equilibrium may exhibit credit rationing even without adverse selection and moral hazard. The literature also identifies various categories of credit rationing of which Freimer and Gordon's (1965) weak and strict rationing is one categorization. Another categorization given by Keeton (1979) (as cited in Baltensperger and Devinny, 1985) distinguishes between Type I rationing (also called loan size rationing) in which some borrowers receive only a fraction of the loan amount so that their credit demands are not completely met, and Type II rationing (also called loan quantity rationing) in which some borrowers' credit demands are fully met while some others are denied any credit.

Notwithstanding the large theoretical literature on "what explains" credit rationing, this literature is silent on "how" the lender would/should decide whom to lend and whom to 'ration out' among so-called 'identical' borrowers. Empirical literature, however, has abundantly documented that the 'rationed out' borrowers are generally the poor, the marginalized and persons of underprivileged ethnicities. In the context of the US economy, Avery (1981) analyzed data on low income households in New Jersey and found empirical evidence that black and Hispanic households, who had significantly positive demand for consumer credit had suffered significantly negative supply of such credit compared to comparable white households. Thus, he concluded that *"observed differences in black-white debt holdings were a supply not a demand phenomenon"*, indicating credit rationing of non-white households than comparable white households. Similarly, Japelli (1990) linked existence of credit constrained consumers and their personal characteristics using data from US Survey of Consumer Finances and found that being white significantly reduced the probability of being credit constrained in addition to being high income and wealthy, thus providing evidence that credit rationed borrowers are those who are poorer and marginalized. In the home mortgage market of the USA, lending patterns were found to be largely skewed against black neighbourhoods by Nesiba (1996). Similar patterns were observed in US small business credit market by Blanchflower et al. (2003) and Cavalluzzo et al. (2002) where they found evidence of significantly higher denial of loans to African-American, Hispanic and Asian owned small businesses compared to white owned businesses. In India, individual characteristics like caste, gender, income levels and education levels are considered important for access to bank credit for borrowers, and credit constrained borrowers are found predominantly from lower caste categories, the poorer, the females and the less educated ones (Chaudhury and Cherical, 2012; Nikaido et al., 2015; Raj and Sasidharan, 2018). Similarly, females in low income urban households in Philippines were found more likely to be more credit constrained than males (Malapit, 2012) and female managed firms were found less likely to receive loans in least financially developed countries in Europe (Muravyev et al., 2007). These empirical studies provide evidence that credit rationed borrowers are predominantly from disadvantaged ethnicities (e.g., non-whites in the USA and lower caste groups in India), low and moderate income groups, disadvantaged gender and people who live at the margins of the society. These borrowers not only face difficulty in obtaining credit but also receive less credit (Blanchflower et al., 2003; Muravyev et al. 2007; Raj and Sasidharan, 2018). Credit rationing of the poor, ethnically underprivileged and marginalized borrowers has also been termed as ‘redlining’, which can be defined as a discriminatory practice against economically and racially disadvantaged individuals and households (Dymski, 1995; Nesiba, 1996; Hillier, 2003); redlining is found to be prevalent in many economies characterized by racial or caste-based segmentation of economic agents.

Regulators generally address such credit market outcomes through various intervention policies, such as interest rate subsidy (or credit subsidy), loan guarantee scheme, directed lending policy and so on. The effectiveness of credit market interventions has been of interest to many scholars. Many have argued that such intervention policies are not without costs as they distort the market-based outcomes and potentially lead to less optimal allocation of credit, particularly when credit directed towards certain sectors through intervention results in reduction of credit to other sectors with potentially higher returns (World Bank, 1989; Buttari, 1995). Further, it is also argued that even though intervention policies may improve credit flow to underserved sectors, increased flow of credit need not by itself translate into better economic outcomes (Schwarz, 1992; Vittas and Cho, 1995). On the other hand, several empirical studies found credit market intervention policies to be effective and beneficial for the target group. For example, loan guarantee schemes in the US, UK, Central and Eastern Europe, France, Japan and Korea have been found to increase employment, investment, survival of new enterprises and supply of funds (Craig et al., 2008; Kang and Heshmati, 2008; Cowling and Siepel, 2013; Lelarge et al., 2010; Uesugi et al., 2010). Directed lending programs were found to have reduced credit constraints of small firms in India, machine tool producers in Japan and LMI communities and minority borrowers in US (Calomiris and Himmelberg, 1993; Eastwood and Kohli, 1999; Barr, 2005; Banerjee and Duflo, 2014; Yadav and Sarma, 2021).

Theoretical economists too have contributed substantially in analyzing the effectiveness of credit market intervention policies. This literature mainly revolves around various forms of credit market intervention in the USA. An early paper by Ordover and Weiss (1981) examined a simple form of intervention, i.e., “forbidding banks from denying loans to an entire class of borrowers”,[2] and argued that “forcing banks to lend to all borrowers at some interest rate” would

[2] This, according to us, is similar in spirit to the provisions under CRA in the USA.

not only increase banks' average total return per dollar of loan but also mitigate exclusion of borrowers due to credit rationing. Mankiw (1986) presented a more formal model of intervention in the student loan market with asymmetric information where borrowers have better information on default probabilities than the lenders. Using interest rate subsidy as a form of intervention, Mankiw (1986) demonstrated that the equilibrium outcome in an unfettered market is precarious and inefficient which can be improved by intervention, even if the government has no informational advantage over the lenders. Other forms of intervention such as credit guarantee, direct loans and government equity investment grants to rationed borrowers are examined by Smith and Stutzer (1989), who showed that credit guarantee by government to all borrowers leads to an improvement in the probability of credit to rationed out borrowers thus improving borrowers' expected utility (welfare) that also improves economic efficiency measured in terms of expected output of funded projects; however, direct loans and equity investment may result in a trade-off between economic efficiency and rationed borrowers' welfare. Innes (1990) focused on three types of information asymmetry in the agricultural credit market and showed that intervention in the form of interest rate subsidy can improve market equilibrium in a Pareto efficient manner vis-à-vis the equilibrium without intervention in all three situations. On the other hand, using two types of "private information model", viz., a "costly state verification model" and a "costly screening model" in credit markets with imperfect information and credit rationing, Williamson (1994) showed that direct government lending and loan guarantee programme do not improve allocation of credit and can worsen the equilibrium market outcome. Relative merits of interest rate subsidy and loan guarantee is investigated by Latruffe and Fraser (2002) and Janda (2011). If the pool of borrowers is collateral constrained then loan guarantee schemes work better and if the expected utility from the loan is below the reservation utility than interest rate subsidies work better as demonstrated through a set of simulation/numerical exercises by Latruffe and Fraser (2002). Janda (2011) found that both these intervention policies have an unambiguous positive effect on social efficiency and such policies benefit all borrowers and lenders. From budgetary consideration, credit guarantees are favorable when there is heterogeneity in project success whereas interest rate subsidies are preferred when there is less project diversity because subsidies are paid to all irrespective of the success or failure whereas guarantee is paid only in case of failures (Janda, 2011). In Rai (2007), government loan programme with co-financing is used as a form of credit market intervention in a market with credit rationing where it was shown that if government is introduced as a co-financier with the private lender then government intervention is beneficial in reducing credit rationing and increasing the expected utility of the borrowers when the government is the first claimant to loan repayment.

To summarize, credit rationing is a commonly observed phenomenon and theoretical examination of it concludes that, in the context of credit market with imperfect information, credit rationing may occur as an equilibrium situation, though an inefficient or a second best one. Theoretical economists interpret imperfect information in terms of lenders' inability to observe borrowers' project quality (ex-ante information asymmetry) and moral hazard (borrower not working hard ex-post or borrowers not truthfully revealing the project outcomes). Given that the overwhelming proportion of credit rationed borrowers are from disadvantaged racial, caste, income and gender categories, a related literature, viz., the redlining literature, considers credit rationing of such borrowers a purely discriminatory practice. Government intervention policies that attempt to mitigate credit rationing/discriminatory lending are widely observed in many economies and their merits and demerits are debated through empirical and theoretical arguments. The benefits of such intervention policies have been observed through both empirical findings as well as theoretical models, although the opposite has also been argued. The theoretical literature on effect of credit market intervention policies in mitigating credit rationing is primarily US-centric, that ignores an important policy prevalent in developing economies, eg., the credit-quota based directed lending policy. In this paper we attempt to fill this gap and present a theoretical model of credit quota based directed lending policy and its effect on credit market outcomes and social welfare. In the following sections, we present our theoretical analysis.

3. A THEORETICAL MODEL OF DIRECTED LENDING

3.1 *Background*

Consider a credit market with two distinguishable sectors, viz., the underserved (also termed marginalized or LMI) sector and the mainstream sector. The underserved sector comprises primarily of racially, ethnically or gender-wise disadvantaged persons, poor and marginalized individuals. From the lenders' point of view, the underserved sector, by its very nature, is informationally opaque whereby its creditworthiness is not clearly observed unless lenders invest in gathering necessary information, which is costly; hence borrowers from this sector are often rationed out. Exclusion of these borrowers from the credit market hinders build-up of their credit history that further alienates them from the credit market. The mainstream sector, on the other hand, is characterized by more formal nature of business activities comprising of agents who generally belong to the mainstream or more privileged segments of the society. Borrowers from the mainstream sector, due to the nature of the sector are perceived more transparent by the lender than those of the marginalized sector; hence they receive bulk of the credit.

Such duality of sectors is observed in many economies. For example, in the USA, the underserved/marginalized sector may be thought of as LMI or minority neighbourhoods in the context of mortgage lending and small business credit (eg., Nesiba, 1996; Blanchflower et al., 2003). In India, the underserved sector may be thought of as the so-called "priority sectors" such as agriculture, small firms, firms owned by historically disadvantaged caste categories and so on.[3] While the two sectors are characterized by varying levels of transparency from lenders' perspective, it is generally the case that the marginalized sector is capable of good business projects if credit is available. For example, a back of the envelope analysis of India's latest survey (2015-16, 73rd round) of non-agricultural firms by National Sample Survey Office (NSSO, India) indicates that among similarly performing firms (measured by having a gross value added above the sample median), 54 per cent were owned by historically disadvantaged caste categories, thus indicating that marginalized sector is capable of good business projects. Among the similarly performing firms, about 9 per cent were credit constrained; and among the credit constrained firms, 55 per cent were owned by historically disadvantaged caste categories.[4] This example simply indicates that the marginalized sector is as capable but is more credit constrained compared to the mainstream sector, indicating that lenders often ration credit to them.

Given this background, our theoretical model attempts to incorporate a representative profit maximizing lender's credit allocation problem to two borrowers having identical projects, each

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[3] Often the underserved/marginalized/LMI sector is characterized by business activities that are household based or informal in nature.

[4] In the interest of brevity we are not providing details of this estimation; however they can be obtained upon request.

belonging to one of these two sectors. The various components of our model are discussed below.

The Borrowers: Consider two risk-neutral borrowers, indexed by type 1 and type 2. Type 1 borrower is from the underserved/marginalized/LMI sector and while type 2 borrower is from the mainstream sector. Due to the nature of the two sectors, the lender perceives the type 1 borrower as informationally opaque compared to the type 2 borrower. Each borrower type can undertake a risky project which yields Y_i in case of success and 0 in the case of failure (i=1,2). Since the type 1 borrower lacks credit history (making it difficult for the lender to ascertain their creditworthiness), the lender can obtain necessary information on them by incurring certain cost. It is assumed that the lender can observe the probability of project success for the type 1 borrower only after incurring a screening or information gathering cost c per unit of loan while the probability of success for the type 2 borrower can be observed without incurring any cost. Suppose that, after having incurred the cost of information on type 1 borrower, the probability of success is found to be same for both the borrowers, given by δ ($0 < \delta < 1$).[5] Thus, both borrowers have an identically risky project. The only distinction between the borrowers is characterized by the heterogenous sectors they belong to, and this distinction is captured in the model through the parameter c, the cost (per unit of loan) of obtaining information on the type 1 borrower.

Production function: The output Y_i from the project follows a Cobb-Douglas production function given by $Y_i = Al_i^{\alpha}$, where l_i is the amount of loan received by the borrower i (i=1,2), $0<\alpha<1$ and the constant $A>1$ captures the combined effect of all other inputs (labour, machinery, fixed capital etc.) on output. The parametric specification for α implies a decreasing returns to scale of loans on production function. The justification is that loans do not get directly translated into output unlike factors such as labour or machines. Since loans are an intermediary used to procure more direct physical inputs in the production process, there is some loss in the process of transforming loans into direct or physical inputs.

Borrower's Utility function: The i^{th} (i=1,2) borrower invests the loan amount l_i (for which an interest rate r_i is charged) in a risky project that generates a return Y_i if the project is

[5] One can also consider the case when the probability of success of type 1 borrowers, δ_1, say, is different from that of type 2 borrower δ_2. For the sake of simplicity, we consider the case of equal probability, which is in line with the definition of credit rationing where among identical borrowers some receive credit and some do not. If $\delta_1 < \delta_2$, then the lender will be justified not to lend to the type 1 borrower.

successful and 0 if it is unsuccessful, the probability of success being δ. If successful, the borrower repays the loan amount with interest, otherwise there is no need to repay as the borrower enjoys limited liability clause. The i^{th} borrower's expected return is given by

$$U_i = \delta(Y_i - (1 + r_i)l_i) + (1 - \delta) * 0 = \delta(Y_i - (1 + r_i)l_i) \qquad (1)$$

The i^{th} borrower will participate in the credit market only if this expected utility is non-negative. This is known as participation constraint of the i^{th} borrower.

The lender: The representative lender's role is to channelize funds from depositors to be lent to borrowers and earn profit through this intermediation. The lender can obtain funds from the depositors at a unit cost of ρ ($0< \rho<1$). In order to earn interest income, the lender must charge an interest rate higher than ρ to its borrowers.

Lender's Profit function: The lender receives interest income in the case of successful project and loses the amount lent in case of failure. The lender's unit cost of raising funds is ρ. In addition, the lender incurs a cost of c per unit of loan to gather information on type 1 borrower. The expected profit of the lender is given by the following function:

$$\pi = \delta[(1 + r_1)l_1 + (1 + r_2)l_2] - (1 + \rho)(l_1 + l_2) - cl_1 \qquad (2)$$

Loan contract: The lender offers a loan contract that consists of a pair (l_i, r_i) where l_i is the loaned amount and r_i is the interest rate offered to the i^{th} borrower (i=1,2). The optimal loan contracts are derived by maximizing lender's expected profit subject to fulfilling each borrower's participation constraint.

Directed lending policy and Social welfare: The regulator wishes to ensure that the type 1 borrower receive at least β proportion of the total loanable funds ($0< \beta <1$). Thus, the lender is mandated to supply at least β proportion of credit to type 1 borrower. While doing so, the regulator aspires to maximize social welfare which is defined as the sum of lender's expected profit plus total utility of the two borrowers, i.e., $\pi + U_1 + U_2$, where π is given by equation (2) and U_i (i=1,2) is given by equation (1).

3.2. Case 0 (Baseline Model) – Both Borrowers from the Mainstream Sector

To begin with, it may be insightful to consider a baseline scenario where there is only one sector, viz., the mainstream sector and both borrowers are from the mainstream sector. This is a situation where the two borrowers are identical in all aspects – in their project risk as well as in their level of information transparency; hence the lender will not incur any cost for information gathering and verification of any of the borrower. In this case, the expected profit of the lender will not have the term involving c in equation (2). The optimal loan contract will be the one where the lender maximizes their expected profit subject to the participation constraints of the borrowers. This is given by Problem P0 below.

$$\max\ \pi(r_1, r_2, l_1, l_2) = \delta[(1 + r_1)l_1 + (1 + r_2)l_2] - (1 + \rho)(l_1 + l_2) \qquad \text{(P0)}$$

$$\text{subject to}$$

$$\delta(Y_1 - (1 + r_1)l_1) \geq 0$$

$$\delta(Y_2 - (1 + r_2)l_2) \geq 0$$

$$r_1, r_2, l_1, l_2 \geq 0$$

The optimal solution of this problem is given by $(r_1^{(0)}, l_1^{(0)}), (r_2^{(0)}, l_2^{(0)})$ below:

$$r_1^{(0)} = r_2^{(0)} = \frac{1 + \rho}{\delta\alpha} - 1$$

$$l_1^{(0)} = l_2^{(0)} = \left[\frac{A\delta\alpha}{1 + \rho}\right]^{\frac{1}{1-\alpha}}$$

Proof: See Appendix A.

The above solution indicates that in this case, the lender offers identical loan contract to both borrowers as they are identical in all aspects. The expression for optimal levels of interest rate and loan amounts are intuitive. Interest rate is a decreasing function of probability of project success (δ) and elasticity of output (α) in the borrower's production function, and an increasing function of lender's cost of funds (ρ). Additionally, as evident from the above expressions, the interest rate and loan amount are inversely related and their functional relationship is a convex function. Thus, the lender offers less amount of loan with a rise in the interest rate and the rate of reduction in loan amount is higher than the rate at which interest rate increases. Following Hodgman (1960), we interpret this inverse relation as lender's precaution towards the fact that increasing interest rate increases volume of repayment which potentially increases borrowers' risk of default. Thus, with high interest rate, a lower volume of loan is supplied and with low interest rate, a higher loan amount is supplied to the borrower. This baseline scenario identifies a case in which both borrowers receive the same loan contract and the loanable fund is divided between the two equally.

3.3. Case 1- Type 1 Borrower from Underserved Sector, No Government Intervention

Now consider the case when the type 1 borrower is perceived less transparent than the type 2 borrower due to the nature of the sector they belong to, even though they have identical projects revealed by their loan applications. The lender must gather and verify information about type 1 borrower by incurring a unit cost of c per unit of loan amount. There is no information opaqueness and hence no additional cost of information gathering as far as type 2 borrower is concerned. The lender's optimization exercise is as follows

$$\max\ \pi(r_1, r_2, l_1, l_2) = \delta[(1+r_1)l_1 + (1+r_2)l_2] - (1+\rho)(l_1+l_2) - cl_1 \qquad \text{(P1)}$$

Subject to

$$\delta(Y_1 - (1+r_1)l_1) \geq 0$$

$$\delta(Y_2 - (1+r_2)l_2) \geq 0$$

$$r_1, r_2, l_1, l_2 \geq 0$$

The optimal solution of the associated problem is given by $(r_1^{(1)}, l_1^{(1)}), (r_2^{(1)}, l_2^{(1)})$:

$$r_1^{(1)} = \frac{1+\rho+c}{\delta\alpha} - 1 \qquad r_2^{(1)} = \frac{1+\rho}{\delta\alpha} - 1$$

$$l_1^{(1)} = \left[\frac{A\delta\alpha}{1+\rho+c}\right]^{\frac{1}{1-\alpha}} \qquad l_2^{(1)} = \left[\frac{A\delta\alpha}{1+\rho}\right]^{\frac{1}{1-\alpha}}$$

Proof: See Appendix B

As seen above, in this case, the lender offers two different loan contracts to the two types of borrowers. Although the loan contract of the type 2 borrower remains the same as the baseline case (case 0), the type 1 borrower is offered lower loan amount, that too at a higher interest rate. The premium on interest rate is on account of the extra screening costs incurred, which is passed on to the type 1 borrower completely. The inverse relationship between interest rate and loan amount implies that while the interest rate charged to type 1 borrower is higher, the loan disbursement to them is lower, thus, making the loan contract offered to type 1 borrower twice as unfavorable. In the extreme case where screening costs increase infinitely, type 1 borrower is denied credit (also known as red-lining). Thus, in this case, type 1 borrower is either "quantity rationed" in the extreme case of infinite screening cost, or "loan size rationed" when the screening cost is finite. On the other hand, the type 2 borrower's loan contract remains the same as the baseline case.

3.4. Case 2 - Government Intervention in the form of Directed Lending Policy

Through the directed lending policy, the regulator mandates that the lender offers at least β proportion of their total loanable funds to the type 1 borrower. This introduces an additional constraint in the optimization problem faced by the lender. The constraint is given by:

$$\frac{l_1}{l_1 + l_2} \geq \beta$$

where $0 \leq \beta \leq 1$. The modified maximization problem of the lender under this intervention is as follows:

$$\max\ \pi(r_1, r_2, l_1, l_2) = \delta[(1 + r_1)l_1 + (1 + r_2)l_2] - (1 + \rho)(l_1 + l_2) - cl_1 \qquad \text{(P2)}$$

Subject to

$$\delta(Y_1 - (1 + r_1)l_1) \geq 0$$

$$\delta(Y_2 - (1 + r_2)l_2) \geq 0$$

$$\frac{l_1}{l_1 + l_2} \geq \beta$$

$$r_1, r_2, l_1, l_2 \geq 0$$

The solution $(r_1^{(2)}, l_1^{(2)}), (r_2^{(2)}, l_2^{(2)})$ to this problem is as follows:

$$r_1^{(2)} = \left(\frac{1 + \rho + \beta c}{\delta\alpha}\right)\left(\frac{\beta^{\alpha-1}}{(1-\beta)^{\alpha} + \beta^{\alpha}}\right) - 1 \qquad r_2^{(2)} = \left(\frac{1 + \rho + \beta c}{\delta\alpha}\right)\left(\frac{(1-\beta)^{\alpha-1}}{(1-\beta)^{\alpha} + \beta^{\alpha}}\right) - 1$$

$$l_1^{(2)} = \left[\left(\frac{A\delta\alpha}{1 + \rho + \beta c}\right)\left(\frac{(1-\beta)^{\alpha} + \beta^{\alpha}}{\beta^{\alpha-1}}\right)\right]^{\frac{1}{1-\alpha}} \qquad l_2^{(2)} = \left[\left(\frac{A\delta\alpha}{1 + \rho + \beta c}\right)\left(\frac{(1-\beta)^{\alpha} + \beta^{\alpha}}{(1-\beta)^{\alpha-1}}\right)\right]^{\frac{1}{1-\alpha}}$$

Proof: See Appendix C

In this case, as shown above, it is optimal for the lender to pass on the screening cost to both borrowers and the optimal loan contracts depend substantially on the "credit quota" parameter β, ceteris paribus. It is easy to observe that when β=0.5, both borrowers receive an identical loan contract and there is no credit rationing. Whether the intervention of directed lending policy helps the type 1 borrower can be analyzed by looking at the contracts offered to them when there is no intervention, i.e., $(r_1^{(1)}, l_1^{(1)})$, and the contract offered under the directed lending policy with credit quota, i.e., $(r_1^{(2)}, l_1^{(2)})$. We present the following proposition.

Proposition 1: *Directed lending policy can mitigate credit rationing of marginalized borrower through a credit quota β that lies in the parameter space (0,0.5]. Within this parameter space, the rationing of LMI borrower is mitigated if the value of β satisfies the following condition (3).*

$$\beta > \frac{1}{\left(\frac{1+\rho+c}{1+\rho}\right)^{\frac{1}{(1-\alpha)}}+1} \tag{3}$$

Proof: Provided in Appendix D.

The right hand side of condition (3) is always less than unity. The first term on the denominator of the right side is the ratio of lender's cost to LMI borrower to that to mainstream borrower, raised to the power $\frac{1}{(1-\alpha)}$. This relative cost ratio plays out in an important way in determining the effectiveness of the credit quota policy for the type 1 or LMI borrower. As the relative cost ratio is primarily driven by the parameter c, following cases may be considered.

I. When c=0, the relative cost ratio is 1 and the above condition boils down to $\beta > 0.5$, which indicates that more of the loan amount is directed towards type 1 borrower. This will surely improve the situation of type 1 borrower compared to the earlier situation of no intervention. However, note that the situation of having a zero screening cost (c=0) corresponds to our baseline model, in which the two borrowers are identical in all aspects and there would be no credit rationing and the total loan amount would be equally divided between the borrowers at equal interest rate, as shown through the solutions of the baseline model. Thus, fixing $\beta > 0.5$ will unnecessarily hurt the type 2 borrower; hence the case $\beta > 0.5$ is unnecessary.

II. When the screening cost c is infinite, then the above condition boils down to $\beta > 0$. In this case, any non-negative credit quota, however small, would mitigate credit rationing of type 1 borrower. Thus, we see that for $c \in (0, \infty)$, the above condition is equivalent to $\beta \in (0,0.5)$, and for such values of β, the type 1 borrower receives higher loan amount at lower interest rate when the directed lending policy is introduced vis-à-vis when there is no such intervention.

III. It is interesting to note that under the above condition, the type 2 borrower's loan contract deteriorates with the introduction of directed lending vis-à-vis the case without intervention, indicating the redistributive effect of directed lending policy. However, even under the intervention, type 2 borrower still receives higher amount of loan compared to type 1 borrower when $\beta < 0.5$.

IV. When $\beta = 0.5$, both borrowers receive an identical loan contract and there is no credit rationing of type 1 borrower.

To conclude, any credit quota in the interval (0,0.5] will mitigate credit constraint faced by the marginalized borrower; when $\beta < 0.5$, there is loan size rationing but no redlining while when $\beta = 0.5$, there is neither loan size rationing nor redlining. The policy leads to a perverse outcome when $\beta > 0.5$.

Lender's financial viability: Intervention policies in the credit market have been often criticized as these policies interfere with rational lender's objective function and thus may lead to financial losses. In order to assess whether directed lending policy considered here leads to a loss for the lender, we can examine the optimized expected profit function of the lender under the directed lending policy, which is as follows:[6]

$$\Pi = A^{\frac{1}{1-\alpha}}\delta^{\frac{1}{1-\alpha}}\,\alpha^{\frac{\alpha}{1-\alpha}}\,(1-\alpha)(1+\rho+\beta c)^{\frac{-\alpha}{1-\alpha}}\,[(1-\beta)^{\alpha}+\beta^{\alpha}]^{\frac{1}{1-\alpha}} \qquad (4)$$

It can be seen that the above expected profit of the lender is strictly positive regardless of the value of the credit-quota parameter β, keeping other parameters ceteris paribus. Thus, directed lending policy does not lead to generate negative profits to the lender. This is because even the marginalized borrowers are worthy borrowers.

4. WELFARE ANALYSIS

4.1. Welfare Function

Through the directed lending policy with credit quota β, the regulator wishes to maximize social welfare defined as the sum of lender's expected profit plus the expected utilities of the borrowers. Welfare function $W(\beta)$ is given below:

$$W(\beta) = \delta(1+r_1)l_1 + \delta(1+r_2)l_2 - (1+\rho)(l_1+l_2) - cl_1 + \delta(Y_1 - (1+r_1)l_1) + \delta\,(Y_2 - (1+r_2)l_2)$$
$$= \delta(1+r_1)l_1 + \delta(1+r_2)l_2 - (1+\rho)(l_1+l_2) - cl_1 + \delta(Al_1^{\alpha} - (1+r_1)l_1) + \delta\,(Al_2^{\alpha} - (1+r_2)l_2)$$

Using $1 + r_1 = R_1$, $1 + r_2 = R_2$, $l_1 = \beta L$, and $l_2 = (1-\beta)L$. Assuming total loanable fund to be of unit volume, for simplicity, we can write L=1 without loss of generality. Then

$$W(\beta) = \delta A\left[(\beta)^{\alpha} + \left((1-\beta)\right)^{\alpha}\right] - (1+\rho) - c\beta \qquad (5)$$

[6] Inserting the solution of (P2) in lender's expected profit function we get the expression (4).

It can be shown that W(β) is concave in β.[7] The concave nature of the function implies that it is possible to find an optimal β that maximizes the welfare function. Before we derive the optimality condition in the next subsection, we present in Figure 1 the plots of the welfare function simulated for various hypothesized values of the set of parameters (α, ρ, δ, c). In all these simulations, we have kept A=2.

(Figure 1 here)

In each one of the four panels in Figure 1, we simulate the welfare function as a function of β by keeping three of the parameters (α, ρ, δ, c) fixed and varying the fourth parameter. For example, in Panel A, we compute the welfare function corresponding to three different values of c while keeping α, ρ, δ fixed. In Panel B, we compute the welfare function for three different values of the parameter ρ while keeping values of (α, c, δ) fixed; Panels C and D similarly present respectively the plots of the welfare functions computed for changing α for fixed values of (c, ρ, δ) and that computed for changing levels of δ with fixed levels of the other three parameters (α, ρ, c). In all these various simulations of the welfare function, we clearly visualize the concavity of the welfare function, indicating the existence of a unique maximum. In Panel A, we observe that an increase in lender's screening cost (c) of LMI borrower, while keeping other parameters at a fixed level, decreases level of β that maximizes welfare. An increase in c, ceteris paribus, also leads to a decrease in maximal value of the welfare. Thus screening cost plays an important role in welfare effects of intervention. A similar pattern emerges from Panel B where lender's cost of raising deposits (ρ) has been changed from 0.25 to 0.6, keeping other parameters fixed. As ρ increases, the welfare decreases gradually, decreasing the β level that generates the maximum welfare and also decreasing the maximal welfare level. When the output elasticity parameter α increases, ceteris paribus, a similar pattern is displayed by the welfare function, wherein the optimal level of β as well as the corresponding level of optimum welfare decreases with an increase in α, as shown in Panel C. A different pattern emerges for the welfare function when probability of project success, δ, increases, ceteris paribus, as depicted in Panel D. In this case, the optimal value of credit quota parameter increases as δ increases; further, higher the δ, higher the optimal level of welfare, as depicted by Panel D of Figure 1.

[7] Appendix E presents a proof of the concavity of the welfare function.

To summarize, we observe the following general patterns from these figures – (i) the welfare function has a unique maximum, (ii) the welfare maximizing level of β and the corresponding optimal level of welfare depends on the existing economic conditions, characterized by the exogenous parameters, and (iii) the welfare maximizing level of β is less than 0.5 with β > 0.5 leading to a decrease in welfare. The following subsection provides mathematical results pertaining to these observations.

4.2. Optimal credit-quota

The optimal level of credit quota β is derived by maximizing regulator's welfare function W(β) given in equation (5). We present the following proposition

Proposition 2: *The optimal level of welfare maximizing β is given by the following condition*

$$\beta^{\alpha-1} - (1-\beta)^{\alpha-1} = \frac{c}{A\delta\alpha} \qquad (6)$$

Proof: See Appendix E.

Optimal level of credit quota is a function of the exogenous parameters of the model. Thus, regulator would need to take cognisance of the prevailing economic parameters while fixing the credit quota level. In Figure 2, we provide an illustration of the choice of optimal credit quota. The dotted graph in Figure 2 represents the left side of equation (6) while the horizontal solid line represents the right side of it. The point of intersection gives the optimal level of credit quota.

(Figure 2 here)

The optimal credit quota β depends crucially on the parameter c that distinguishes type 1 borrower from type 2 borrower. We consider the following cases:

Case 1: $c = 0$; in this case, the two borrowers are identical in all aspects, and from (6) we have

$$\beta^{\alpha-1} - (1-\beta)^{\alpha-1} = 0$$

$$\beta = \frac{1}{2}$$

Case 2: $c > 0$; in this case

$$\beta^{\alpha-1} - (1-\beta)^{\alpha-1} > 0$$

$$\beta^{\alpha-1} > (1-\beta)^{\alpha-1}$$

$$\beta < 1-\beta \text{ (since } \alpha < 1)$$

$$\beta < \frac{1}{2}$$

Thus, we conclude that for $c > 0$, the optimal parameter space for β can be fixed at a value within the range $\left(0, \frac{1}{2}\right)$. As shown in Proposition 1, such a level of β, however small, improves the loan contract of type 1 borrower compared to the case without any intervention. The case $\beta < \frac{1}{2}$ eliminates redlining although loan size to type 1 borrower will be less than that offered to type 2 borrower. Even then, compared to the case without any intervention, type 1 borrower receives higher loan amount in this case; thus making intervention beneficial.

To summarize, directed lending policy with credit quota in (0, 0.5) is able to mitigate credit rationing of the marginalized borrowers through a redistribution of total loanable funds. When the credit quota is less than 0.5, the marginalized borrower still faces some amount of "loan size rationing", but the directed lending policy removes the possibility of redlining (also known as quantity rationing) of the marginalized borrower. In addition, their terms of loan contract improve vis-à-vis the situation without intervention. This is an important first step towards initiating build-up of credit history for the marginalized borrower. Further, introduction of credit quota policy does not generate negative profit for the lender, as lender's expected profit function is found to be strictly positive for all values of β. The total welfare function of the regulator is concave in β implying that it is possible to derive an optimal β that maximizes the total welfare of the economy.

4.3. Comparative Statics

The optimality condition (6) shows that the optimal credit quota depends on exogeneous parameters (c, δ, A). Taking total differential in (6), we have the following results:

i. $\frac{d\beta}{dc} < 0$, ceteris paribus. Thus, as screening costs increases, ceteris paribus, the welfare maximizing optimal credit quota can be fixed at a lower level.

ii. $\frac{d\beta}{d\delta} > 0$, ceteris paribus. Thus project success probability and welfare maximizing credit quota level move in the same direction, keeping other parameters fixed.

iii. $\frac{d\beta}{dA} > 0$, ceteris paribus. Thus improvement in production technology, keeping other parameters fixed, leads to increase in optimal β.

Proof: See Appendix F.

These results are consistent with the patterns displayed through FIGURE I and highlight that while deriving the optimal credit quota, the regulator must take cognizance of the existing economic conditions characterized by various exogenous parameters.

5. CONCLUSION

The extant theoretical literature on credit market intervention policies has not yet considered the directed lending policy, an important intervention policy prevalent in many economies. In an attempt to fill this gap, we present here a simple model of credit-quota based directed lending policy and examine welfare implications of such a policy. When an economy comprises of marginalized and mainstream sectors, credit quota based directed lending policy can mitigate redlining and credit rationing of marginalized borrowers. We find that as long as the credit quota is below the threshold level of 0.5, the directed lending policy is welfare improving for the overall economy that includes borrowers from both sectors as well as the lender. A credit quota higher than 0.5 may reduce welfare. Further, the optimal level of credit quota is a function of existing economic conditions characterized by exogenous parameters that characterize the economy. The important policy implication of our analysis is that credit quota based directed lending policies can be a useful tool for supplying credit to underserved sectors and enhance equality of opportunity.

APPENDIX A

Lender's optimization problem (P0) is

$$\max\ \pi(r_1, r_2, l_1, l_2) = \delta[(1 + \mathrm{r}_1)\mathrm{l}_1 + (1 + \mathrm{r}_2)\mathrm{l}_2] - (1 + \rho)(\mathrm{l}_1 + \mathrm{l}_2)$$

$$= \delta(r_1 l_1 + r_2 l_2) - (1 - \delta + \rho)(l_1 + l_2)$$

Subject to

$$-\delta(Y_1 - (1 + r_1)l_1) \leq 0 \Rightarrow -\delta[A{l_1}^{\alpha} - (1 + r_1)l_1] \leq 0$$

$$-\delta(Y_2 - (1 + r_2)l_2) \leq 0 \Rightarrow -\delta[A{l_2}^{\alpha} - (1 + r_2)l_2] \leq 0$$

$$r_1, r_2, l_1, l_2 \geq 0$$

Kuhn-Tucker Lagrangian:

$$\mathcal{L} = \delta(r_1 l_1 + r_2 l_2) - (1 - \delta + \rho)\,(l_1 + l_2) - \lambda_1(-\delta[A{l_1}^{\alpha} - (1 + r_1)l_1])$$
$$- \lambda_2(-\delta[A{l_2}^{\alpha} - (1 + r_2)l_2])$$

Kuhn-Tucker (K-T) Conditions:

$$\frac{\partial \mathcal{L}}{\partial r_1} \leq 0 \Rightarrow \delta l_1 - \lambda_1 \delta l_1 = \delta(1-\lambda_1) l_1 \leq 0 \ldots\ldots\ldots\ldots\ldots\ldots..(i)$$

$$\frac{\partial \mathcal{L}}{\partial r_2} \leq 0 \Rightarrow \delta l_2 - \lambda_2 \delta l_2 = \delta(1-\lambda_2) l_2 \leq 0 \ldots\ldots\ldots\ldots\ldots\ldots..(ii)$$

$$\frac{\partial \mathcal{L}}{\partial l_1} \leq 0 \Rightarrow \delta r_1 - (1-\delta+\rho) + \lambda_1 \delta[\alpha A l_1^{\alpha-1} - (1+r_1)] \leq 0 \ldots\ldots\ldots.(iii)$$

$$\frac{\partial \mathcal{L}}{\partial l_2} \leq 0 \Rightarrow \delta r_2 - (1-\delta+\rho) + \lambda_2 \delta[\alpha A l_2^{\alpha-1} - (1+r_2)] \leq 0 \ldots\ldots\ldots..(iv)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} \geq 0 \Rightarrow \delta[A l_1{}^{\alpha} - (1+r_1) l_1] \geq 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(v)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} \geq 0 \Rightarrow \delta[A l_2{}^{\alpha} - (1+r_2) l_2] \geq 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(vi)$$

$$r_1 \frac{\partial \mathcal{L}}{\partial r_1} = 0 \Rightarrow r_1 \delta(1-\lambda_1) l_1 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(vii)$$

$$r_2 \frac{\partial \mathcal{L}}{\partial r_2} = 0 \Rightarrow r_2 \delta(1-\lambda_2) l_2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(viii)$$

$$l_1 \frac{\partial \mathcal{L}}{\partial l_1} = 0 \Rightarrow l_1\left[\delta r_1 - (1-\delta+\rho) + \lambda_1 \delta[\alpha A l_1^{\alpha-1} - (1+r_1)]\right] = 0 \ldots\ldots\ldots\ldots\ldots.(ix)$$

$$l_2 \frac{\partial \mathcal{L}}{\partial l_2} = 0 \Rightarrow l_2\left[\delta r_2 - (1-\delta+\rho) + \lambda_2 \delta[\alpha A l_2^{\alpha-1} - (1+r_2)]\right] = 0 \ldots\ldots\ldots\ldots\ldots.(x)$$

$$\lambda_1 \frac{\partial \mathcal{L}}{\partial \lambda_1} = 0 \Rightarrow \lambda_1 \delta[A l_1{}^{\alpha} - (1+r_1) l_1] = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(xi)$$

$$\lambda_2 \frac{\partial \mathcal{L}}{\partial \lambda_2} = 0 \Rightarrow \lambda_2 \delta[A l_2{}^{\alpha} - (1+r_2) l_2] = 0 \ldots\ldots\ldots\ldots\ldots.(xii)$$

Assuming non-zero solutions for (r_1,r_2,l_1,l_2), and using $\delta \epsilon (0,1)$, we have, from (vii) and (viii) above $\lambda_1 = 1;\ \lambda_2 = 1$

Using $\lambda_1 = 1$ in (ix), and $\lambda_2 = 1$ in (x), we get, after simplification, $l_1 = l_2 = \left(\frac{\delta A \alpha}{1+\rho}\right)^{\frac{1}{(1-\alpha)}}$

From (xi) and (xii), it is easy to derive that $r_1 = r_2 = \frac{1+\rho}{\delta\alpha} - 1$

Second Order Condition

The bordered Hessian for the problem is:

$$\overline{\mathrm{H}} = \begin{bmatrix} 0 & 0 & \delta l_1 & 0 & -\delta A\alpha l_1^{\alpha-1} + \delta(1+r_1) & 0 \\ 0 & 0 & 0 & \delta l_2 & 0 & -\delta A\alpha l_2^{\alpha-1} + \delta(1+r_2) \\ \delta l_1 & 0 & 0 & 0 & \delta(1-\lambda_1) & 0 \\ 0 & \delta l_2 & 0 & 0 & 0 & \delta(1-\lambda_2) \\ -\delta A\alpha l_1^{\alpha-1} + \delta(1+r_1) & 0 & \delta(1-\lambda_1) & 0 & \lambda_1\delta\alpha A(\alpha-1)l_1^{\alpha-2} & 0 \\ 0 & -\delta A\alpha l_2^{\alpha-1} + \delta(1+r_2) & 0 & \delta(1-\lambda_2) & 0 & \lambda_2\delta\alpha A(\alpha-1)l_2^{\alpha-2} \end{bmatrix}$$

Evaluate at $\lambda_1 = \lambda_2 = 1$ and using $(1 + r_1) = Al_1^{\alpha-1}$ and $(1 + r_2) = Al_2^{\alpha-1}$

$$\overline{\mathrm{H}} = \begin{bmatrix} 0 & 0 & \delta l_1 & 0 & (1-\alpha)\delta A l_1^{\alpha-1} & 0 \\ 0 & 0 & 0 & \delta l_2 & 0 & (1-\alpha)\delta A l_2^{\alpha-1} \\ \delta l_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta l_2 & 0 & 0 & 0 & 0 \\ (1-\alpha)\delta A l_1^{\alpha-1} & 0 & 0 & 0 & (\alpha-1)\delta\alpha A l_1^{\alpha-2} & 0 \\ 0 & (1-\alpha)\delta A l_2^{\alpha-1} & 0 & 0 & 0 & (\alpha-1)\delta\alpha A l_2^{\alpha-2} \end{bmatrix}$$

$|\overline{H_6}| = \delta^6\alpha^2(\alpha-1)^2A^2(l_1l_2)^\alpha > 0$

$|\overline{\mathrm{H}_5}| = \delta^5\alpha(\alpha-1)Al_1^{\alpha}l_2^2 < 0$

Since the minors alternate in sign and since the sign of det H_6 is the sign of $(-1)^4=+1$, the solution of the problem is maximizing.

APPENDIX B

Lender's optimization problem (P1) is

$$\max\ \pi(r_1, r_2, l_1, l_2) = \delta(r_1l_1 + r_2l_2) - (1 + \rho - \delta)(l_1 + l_2) - c\ l_1$$

Subject to

$$-\delta(Y_1 - (1 + r_1)l_1) \le 0 \Rightarrow -\delta[Al_1{}^\alpha - (1 + r_1)l_1] \le 0$$

$$-\delta(Y_2 - (1 + r_2)l_2) \le 0 \Rightarrow -\delta[Al_2{}^\alpha - (1 + r_2)l_2] \le 0$$

$$r_1, r_2, l_1, l_2 \ge 0$$

Kuhn-Tucker Lagrangian:

$$\mathcal{L} = \delta(r_1l_1 + r_2l_2) - (1 + \rho - \delta)(l_1 + l_2) - c\ l_1 - \lambda_1(-\delta[Al_1{}^\alpha - (1 + r_1)l_1]) - \lambda_2(-\delta[Al_2{}^\alpha - (1 + r_2)l_2])$$

Kuhn-Tucker Conditions are:

$$\frac{\partial\mathcal{L}}{\partial r_1} = \delta l_1 - \lambda_1\delta l_1 = \delta(1 - \lambda_1)l_1 \le 0 \ldots\ldots\ldots\ldots\ldots\ldots..(i)$$

$$\frac{\partial\mathcal{L}}{\partial r_2} = \delta l_2 - \lambda_2\delta l_2 = \delta(1 - \lambda_2)l_2 \le 0 \ldots\ldots\ldots\ldots\ldots\ldots..(ii)$$

$$\frac{\partial\mathcal{L}}{\partial l_1} = \delta r_1 - (1 + \rho - \delta) - c + \lambda_1\delta\left(A\alpha l_1^{\alpha-1} - (1 + r_1)\right) \le 0 \ldots.(iii)$$

$$\frac{\partial \mathcal{L}}{\partial l_2} = \delta r_2 - (1+\rho-\delta) + \lambda_2 \delta\left(A\alpha l_2^{\alpha-1} - (1+r_2)\right) \leq 0 \ldots\ldots\ldots..(iv)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = \delta\left[\left(Al_1^{\alpha} - (1+r_1)\right)l_1\right] \geq 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (v)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = \delta\left[\left(Al_2^{\alpha} - (1+r_2)\right)l_2\right] \geq 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (vi)$$

$$\mathrm{r}_1 \frac{\partial \mathcal{L}}{\partial r_1} = \mathrm{r}_1 \delta(1-\lambda_1) l_1 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(vii)$$

$$\mathrm{r}_2 \frac{\partial \mathcal{L}}{\partial r_2} = \mathrm{r}_2 \delta(1-\lambda_2) l_2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (viii)$$

$$\mathrm{l}_1 \frac{\partial \mathcal{L}}{\partial l_1} = \mathrm{l}_1\left[\delta r_1 - (1+\rho-\delta+c) + \lambda_1 \delta\left(A\alpha l_1^{\alpha-1} - (1+r_1)\right)\right] = 0 \ldots\ldots\ldots\ldots\ldots\ldots..(ix)$$

$$\mathrm{l}_2 \frac{\partial \mathcal{L}}{\partial l_2} = \mathrm{l}_2\left[\delta r_2 - (1+\rho-\delta) + \lambda_2 \delta\left(A\alpha l_2^{\alpha-1} - (1+r_2)\right)\right] = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(x)$$

$$\lambda_1 \frac{\partial \mathcal{L}}{\partial \lambda_1} = \lambda_1 \delta\left[\left(Al_1^{\alpha} - (1+r_1)\right)l_1\right] = 0 \ldots\ldots\ldots\ldots\ldots\ldots.(xi)$$

$$\lambda_2 \frac{\partial \mathcal{L}}{\partial \lambda_2} = \lambda_2 \delta\left[\left(Al_2^{\alpha} - (1+r_2)\right)l_2\right] = 0 \ldots\ldots\ldots\ldots\ldots\ldots..(xii)$$

From equations (vii) and (viii), we get $\lambda_1 = \lambda_2 = 1$ allowing for non-zero solutions. Using $\lambda_1 = 1$ in (xi), we get $Al_1^{\alpha-1} = 1 + r_1$ and using $\lambda_2 = 1$ in (xii) we get $Al_2^{\alpha-1} = 1 + r_2$. From (ix) we get $l_1 = \left(\frac{\delta A\alpha}{1+\rho+c}\right)^{\frac{1}{1-\alpha}}$. Similarly, from (x), we get $l_2 = \left(\frac{\delta A\alpha}{1+\rho}\right)^{\frac{1}{1-\alpha}}$. Using these expressions for l_1 and l_2 in (xi) and (xii), we get $r_1 = \frac{1+\rho+c}{\delta\alpha} - 1$; $r_2 = \frac{1+\rho}{\delta\alpha} - 1$

Second Order Condition is exactly the same as that in Appendix A.

APPENDIX C

Lender's optimization problem in (P2) is as follows:

$$\max \ \pi(r_1, r_2, l_1, l_2) = \delta(r_1 l_1 + r_2 l_2) - (1+\rho-\delta)(l_1 + l_2) - c\, l_1$$

Subject to

$$-\delta[A{l_1}^{\alpha} - (1+r_1)l_1] \leq 0 \ ; -\delta[A{l_2}^{\alpha} - (1+r_2)l_2] \leq 0 \ ; \frac{\beta}{1-\beta} - \frac{l_1}{l_2} \leq 0 \ ; r_1, r_2, l_1, l_2 \geq 0$$

Kuhn-Tucker Lagrangian is

$$\mathcal{L} = \delta(r_1 l_1 + r_2 l_2) - (1+\rho-\delta)(l_1+l_2) - c\, l_1 - \lambda_1(-\delta[A{l_1}^{\alpha} - (1+r_1)l_1]) - \lambda_2(-\delta[A{l_2}^{\alpha} - (1+r_2)l_2]) - \mu\left(\frac{\beta}{1-\beta} - \frac{l_1}{l_2}\right)$$

Kuhn-Tucker Conditions:

$$\frac{\partial \mathcal{L}}{\partial r_1} = \delta l_1 - \lambda_1 \delta l_1 = \delta(1-\lambda_1)l_1 \le 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(i)$$

$$\frac{\partial \mathcal{L}}{\partial r_2} = \delta l_2 - \lambda_2 \delta l_2 = \delta(1-\lambda_2)l_2 \le 0 \ldots\ldots\ldots\ldots\ldots\ldots..\ldots..(ii)$$

$$\frac{\partial \mathcal{L}}{\partial l_1} = \delta(r_1+1) - c - (1+\rho) + \lambda_1\delta\left(A\alpha l_1^{\alpha-1} - (1+r_1)\right) + \frac{\mu}{l_2} \le 0 \ldots.(iii)$$

$$\frac{\partial \mathcal{L}}{\partial l_2} = \delta(r_2+1) - (1+\rho) + \lambda_2\delta\left(A\alpha l_2^{\alpha-1} - (1+r_2)\right) - \frac{\mu l_1}{l_2^2} \le 0 \ldots\ldots..(iv)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = \delta[A{l_1}^{\alpha} - (1+r_1)l_1] \ge 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(v)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = \delta[A{l_2}^{\alpha} - (1+r_2)l_2] \ge 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ldots\ldots\ldots\ldots(vi)$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = -\frac{\beta}{1-\beta} + \frac{l_1}{l_2} \ge 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(vii)$$

$$r_1 \frac{\partial \mathcal{L}}{\partial r_1} = r_1\delta(1-\lambda_1)l_1 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(viii)$$

$$r_2 \frac{\partial \mathcal{L}}{\partial r_2} = r_2\delta(1-\lambda_2)l_2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(ix)$$

$$l_1 \frac{\partial \mathcal{L}}{\partial l_1} = l_1\left[\delta(r_1+1) - c - (1+\rho) + \lambda_1\delta\left(A\alpha l_1^{\alpha-1} - (1+r_1)\right) + \frac{\mu}{l_2}\right] = 0 \ldots\ldots\ldots\ldots\ldots\ldots..(x)$$

$$l_2 \frac{\partial \mathcal{L}}{\partial l_2} = l_2\left[\delta(r_2+1) - (1+\rho) + \lambda_2\delta\left(A\alpha l_2^{\alpha-1} - (1+r_2)\right) - \frac{\mu l_1}{l_2^2}\right] = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(xi)$$

$$\lambda_1 \frac{\partial \mathcal{L}}{\partial \lambda_1} = \lambda_1\delta\left[\left(A l_1^{\alpha} - (1+r_1)\right)l_1\right] = 0 \ldots\ldots\ldots\ldots\ldots\ldots.(xii)$$

$$\lambda_2 \frac{\partial \mathcal{L}}{\partial \lambda_2} = \lambda_2\delta\left[\left(A l_2^{\alpha} - (1+r_2)\right)l_2\right] = 0 \ldots\ldots\ldots\ldots\ldots\ldots..(xiii)$$

$$\mu \frac{\partial \mathcal{L}}{\partial \mu} = \mu\ \left(\frac{l_1}{l_2} - \frac{\beta}{1-\beta}\right) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(xiv)$$

Considering only non-zero solutions,

(viii) ⇒ $\lambda_1 = 1$; (ix) ⇒ $\lambda_2 = 1$; (xii) ⇒ $A l_1^{\alpha-1} = (1+r_1) \Rightarrow l_1^{\alpha-1} = \frac{1+r_1}{A}$; (xiii) ⇒ $l_2^{\alpha-1} = \frac{1+r_2}{A}$

(x) ⇒ $\delta A\alpha l_1 + \frac{\mu}{l_2} = 1+\rho+c \Rightarrow \delta\alpha\,(1+r_1) + \frac{\mu}{l_2} = 1+\rho+c$; (Using $A l_1^{\alpha-1} = 1+r_1$ (from above)

(xi) ⇒ $\delta\alpha\,(1+r_2) - \frac{\mu l_1}{{l_2}^2} = 1+\rho$; (Using $A l_2^{\alpha-1} = 1+r_2$ (from above)

(xiv) ⇒ either $\mu = 0$ or $\frac{l_1}{l_2} = \frac{\beta}{1-\beta}$; The case when $\mu = 0$ implies that the constraint is not binding so that the optimization without the constraint will yield the same solution as the optimization with the constraint. Since this constraint represents the directed lending policy, we want this constraint to be binding and hence assume $\mu \neq 0$ and let $\frac{l_1}{l_2} = \frac{\beta}{1-\beta} \Rightarrow l_1 = \frac{\beta l_2}{1-\beta}$. Then, from (xi), we have

$$\delta\alpha\,(1 + r_2) - \frac{\mu l_1}{{l_2}^2} = 1 + \rho \Rightarrow \delta\alpha\,(1 + r_2) - \frac{\beta}{(1-\beta)}\frac{\mu}{l_2} = 1 + \rho$$

$\Rightarrow \delta\alpha\,(1 + r_2) - \frac{\beta}{(1-\beta)}[1 + \rho + c - \delta\alpha\,(1 + r_1)] = 1 + \rho$; using $\frac{\mu}{l_2} = 1 + \rho + c - \delta\alpha\,(1 + r_1)$

$\Rightarrow \delta\alpha\, A l_2^{\alpha-1} - \frac{\beta}{(1-\beta)}\left[1 + \rho + c - \delta\alpha\, A\left(\frac{\beta l_2}{1-\beta}\right)^{(\alpha-1)}\right] = 1 + \rho$; using $A l_2^{\alpha-1} = 1 + r_2$; $A l_1^{\alpha-1} = 1 + r_1$; $l_1 = \frac{\beta l_2}{1-\beta}$

$$\Rightarrow \delta\alpha\, A l_2^{\alpha-1}\left[1 + \left(\frac{\beta}{1-\beta}\right)^{\alpha}\right] = 1 + \rho + \left(\frac{\beta}{1-\beta}\right)(1 + \rho + c) = \frac{1 + \rho + \beta c}{1-\beta}$$

$\Rightarrow l_2^{\alpha-1} = \left(\frac{1+\rho+\beta c}{\delta\alpha\, A}\right)\left(\frac{(1-\beta)^{\alpha-1}}{(1-\beta)^{\alpha}+\beta^{\alpha}}\right)$ (Upon simplification)

$\Rightarrow\; l_2 = \left(\frac{\delta\alpha\, A}{1+\rho+\beta c}\right)^{\frac{1}{(1-\alpha)}}(1-\beta)[\beta^{\alpha} + (1-\beta)^{\alpha}]^{\frac{1}{(1-\alpha)}} = \left[\left(\frac{A\delta\alpha}{1+\rho+\beta c}\right)\;\left(\frac{(1-\beta)^{\alpha}+\beta^{\alpha}}{(1-\beta)^{\alpha-1}}\right)\right]^{\frac{1}{1-\alpha}}$ (Upon simplification)

$l_1 = \frac{\beta l_2}{1-\beta} = \frac{\beta}{(1-\beta)}\left(\frac{\delta\alpha\, A}{1+\rho+\beta c}\right)^{\frac{1}{(1-\alpha)}}(1-\beta)[\beta^{\alpha} + (1-\beta)^{\alpha}]^{\frac{1}{(1-\alpha)}}$ using the expression for l_2 from above

$\Rightarrow l_1 = \left(\frac{\delta\alpha\, A}{1+\rho+\beta c}\right)^{\frac{1}{(1-\alpha)}}\beta[\beta^{\alpha} + (1-\beta)^{\alpha}]^{\frac{1}{(1-\alpha)}} = \left[\left(\frac{A\delta\alpha}{1+\rho+\beta c}\right)\;\left(\frac{(1-\beta)^{\alpha}+\beta^{\alpha}}{\beta^{\alpha-1}}\right)\right]^{\frac{1}{1-\alpha}}$; on simplification

$$1 + r_1 = A l_1^{\alpha-1} = A\beta^{(\alpha-1)}[\beta^{\alpha} + (1-\beta)^{\alpha}]^{-1}\left(\frac{\delta\alpha\, A}{1 + \rho + \beta c}\right)^{-1} = \frac{(1 + \rho + \beta c)\beta^{(\alpha-1)}}{\delta\alpha[\beta^{\alpha} + (1-\beta)^{\alpha}]}$$

$$\Rightarrow r_1 = \frac{(1 + \rho + \beta c)\beta^{(\alpha-1)}}{\delta\alpha[\beta^{\alpha} + (1-\beta)^{\alpha}]} - 1$$

Following similar steps, we get, by using $A l_2^{\alpha-1} = 1 + r_2$ and expression for l_2 from above

$$r_2 = \frac{(1 + \rho + \beta c)(1-\beta)^{(\alpha-1)}}{\delta\alpha[\beta^{\alpha} + (1-\beta)^{\alpha}]} - 1$$

SOC:

$$\bar{H} = \begin{bmatrix} 0 & 0 & 0 & \delta l_1 & 0 & -\delta A\alpha l_1^{\alpha-1}+\delta(1+r_1) & 0 \\ 0 & 0 & 0 & 0 & \delta l_2 & 0 & -\delta A\alpha l_2^{\alpha-1}+\delta(1+r_2) \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{l_2} & \frac{l_1}{l_2^2} \\ \delta l_1 & 0 & 0 & 0 & 0 & \delta(1-\lambda_1) & 0 \\ 0 & \delta l_2 & 0 & 0 & 0 & 0 & \delta(1-\lambda_2) \\ -\delta A\alpha l_1^{\alpha-1}+\delta(1+r_1) & 0 & -\frac{1}{l_2} & \delta(1-\lambda_1) & 0 & \lambda_1\delta\alpha A(\alpha-1)l_1^{\alpha-2} & -\frac{\mu}{l_2^2} \\ 0 & -\delta A\alpha l_2^{\alpha-1}+\delta(1+r_2) & \frac{l_1}{l_2^2} & 0 & \delta(1-\lambda_2) & -\frac{\mu}{l_2^2} & \lambda_2\delta\alpha A(\alpha-1)l_2^{\alpha-2}+\frac{2\mu l_1}{l_2^3} \end{bmatrix}$$

Evaluate at $\lambda_1 = \lambda_2 = 1$ and using $(1 + r_1) = Al_1^{\alpha-1}$ and $(1 + r_2) = Al_2^{\alpha-1}$

$$\bar{H} = \begin{bmatrix} 0 & 0 & 0 & \delta l_1 & 0 & \delta(1-\alpha)Al_1^{\alpha-1} & 0 \\ 0 & 0 & 0 & 0 & \delta l_2 & 0 & \delta(1-\alpha)Al_2^{\alpha-1} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{l_2} & \frac{l_1}{l_2^2} \\ \delta l_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta l_2 & 0 & 0 & 0 & 0 & 0 \\ \delta(1-\alpha)Al_1^{\alpha-1} & 0 & -\frac{1}{l_2} & 0 & 0 & \delta\alpha A(\alpha-1)l_1^{\alpha-2} & -\frac{\mu}{l_2^2} \\ 0 & \delta(1-\alpha)Al_2^{\alpha-1} & \frac{l_1}{l_2^2} & 0 & 0 & -\frac{\mu}{l_2^2} & \delta\alpha A(\alpha-1)l_2^{\alpha-2}+\frac{2\mu l_1}{l_2^3} \end{bmatrix}$$

$$|\overline{H_7}| = A\delta^5\alpha(1-\alpha)\left(\frac{l_1}{l_2}\right)^2 \{l_1^\alpha + l_2^\alpha\} > 0$$

Since the sign of det $\overline{H_7}$ is the sign of $(-1)^4=+1$, the solution of the problem is maximizing.

APPENDIX D: PROOF OF PROPOSITION 1

If we assume that interest rate for marginalized borrower falls with intervention then,

$$\left[\left(\frac{1+\rho+\beta c}{\delta\alpha}\right)\left(\frac{\beta^{\alpha-1}}{(1-\beta)^\alpha+\beta^\alpha}\right)-1\right]-\left[\frac{1+\rho+c}{\delta\alpha}-1\right] \le 0 \Rightarrow \left(\frac{1+\rho+\beta c}{\delta\alpha}\right)\left(\frac{\beta^{\alpha-1}}{(1-\beta)^\alpha+\beta^\alpha}\right)-\frac{1+\rho+c}{\delta\alpha} \le 0$$

$$\Rightarrow \left(\frac{1+\rho+\beta c}{\delta\alpha}\right)\left(\frac{\beta^{\alpha-1}}{(1-\beta)^\alpha+\beta^\alpha}\right) \le \frac{1+\rho+c}{\delta\alpha} \Rightarrow \left(\frac{1+\rho+\beta c}{1+\rho+c}\right) \le \frac{(1-\beta)^\alpha+\beta^\alpha}{\beta^{\alpha-1}} \qquad \text{---------} \qquad \textbf{(*)}$$

$$\Longrightarrow \beta^{\alpha-1}(1+\rho+\beta c) \le (1-\beta)^\alpha(1+\rho+c)+\beta^\alpha(1+\rho+c)$$

$$\Longrightarrow \beta^{\alpha-1}(1+\rho) \le (1-\beta)^\alpha(1+\rho+c)+\beta^\alpha(1+\rho)$$

$$\Longrightarrow (\beta^{\alpha-1}-\beta^\alpha)(1+\rho) \le (1-\beta)^\alpha(1+\rho+c)$$

$$\Longrightarrow \beta^\alpha(\beta^{-1}-1)(1+\rho) \le (1-\beta)^\alpha(1+\rho+c)$$

$$\Longrightarrow \left(\frac{\beta}{1-\beta}\right)^\alpha\left(\frac{1}{\beta}-1\right) \le \frac{1+\rho+c}{1+\rho}$$

$$\Longrightarrow \left(\frac{\beta}{1-\beta}\right)^\alpha\left(\frac{1-\beta}{\beta}\right) \le \frac{1+\rho+c}{1+\rho} \Rightarrow \left[\frac{1-\beta}{\beta}\right]^{1-\alpha} \le \frac{1+\rho}{1+\rho+c} \Rightarrow \frac{1-\beta}{\beta} \le \left[\frac{1+\rho+c}{1+\rho}\right]^{\frac{1}{1-\alpha}}$$

$$\Rightarrow \frac{1}{\beta} \leq \left[\frac{1+\rho+c}{1+\rho}\right]^{\frac{1}{1-\alpha}} +1 \;\Rightarrow \beta \geq \left[\left(\frac{1+\rho+c}{1+\rho}\right)^{\frac{1}{1-\alpha}}+1\right]^{-1}$$

If we assume that loan amount for marginalized borrower increases with intervention then,

$$\left[\left(\frac{A\delta\alpha}{1+\rho+\beta c}\right)\left(\frac{(1-\beta)^{\alpha}+\beta^{\alpha}}{\beta^{\alpha-1}}\right)\right]^{\frac{1}{1-\alpha}} - \left[\frac{A\delta\alpha}{1+\rho+c}\right]^{\frac{1}{1-\alpha}} \geq 0$$

$\left[\left(\frac{A\delta\alpha}{1+\rho+\beta c}\right)\left(\frac{(1-\beta)^{\alpha}+\beta^{\alpha}}{\beta^{\alpha-1}}\right)\right]^{\frac{1}{1-\alpha}} \geq \left[\frac{A\delta\alpha}{1+\rho+c}\right]^{\frac{1}{1-\alpha}}$; Raising both RHS and LHS to the power $(1-\alpha)$,

we get $\left(\frac{A\delta\alpha}{1+\rho+\beta c}\right)\left(\frac{(1-\beta)^{\alpha}+\beta^{\alpha}}{\beta^{\alpha-1}}\right) \geq \left[\frac{A\delta\alpha}{1+\rho+c}\right] \Rightarrow \left(\frac{1}{1+\rho+\beta c}\right)\left(\frac{(1-\beta)^{\alpha}+\beta^{\alpha}}{\beta^{\alpha-1}}\right) \geq \left[\frac{1}{1+\rho+c}\right]$

Multiply both sides by $1+\rho+\beta c$, we get $\frac{1+\rho+\beta c}{1+\rho+c} \leq \left(\frac{(1-\beta)^{\alpha}+\beta^{\alpha}}{\beta^{\alpha-1}}\right)$

This condition is same as the one marked as (*) above, thus we can apply the same algebra to show that the condition for higher loan amount under intervention compared to the case without any intervention is $\beta \geq \left[\left(\frac{1+\rho+c}{1+\rho}\right)^{\frac{1}{1-\alpha}}+1\right]^{-1}$

APPENDIX E

Welfare function W(β) is defined as the sum total of lender's expected profit and utility of each borrower:

$$W(\beta) = \delta(1+r_1)l_1 + \delta(1+r_2)l_2 - (1+\rho)(l_1+l_2) - cl_1 + \delta(Y_1 - (1+r_1)l_1) + \delta\,(Y_2 - (1+r_2)l_2)$$

$$= \delta(1+r_1)l_1 + \delta(1+r_2)l_2 - (1+\rho)(l_1+l_2) - cl_1 + \delta(Al_1^{\alpha} - (1+r_1)l_1) + \delta\,(Al_2^{\alpha} - (1+r_2)l_2)$$

Using $1+r_1 = R_1$, $1+r_2 = R_2$, $l_1 = \beta L$, and $l_2 = (1-\beta)L$. Assuming total loanable fund to be of unit volume, for simplicity, we can write L=1 without loss of generality. Then

$$W(\beta) = \delta A\left[(\beta)^{\alpha} + \left((1-\beta)\right)^{\alpha}\right] - (1+\rho) - c\beta$$

Proof of concavity of W(β):

$$\frac{dW}{d\beta} = \delta A[\alpha\beta^{\alpha-1} - \alpha(1-\beta)^{\alpha-1}] - c$$

$$\frac{d^2W}{d\beta^2} = \delta A\alpha[(\alpha-1)\beta^{\alpha-2} + (\alpha-1)(1-\beta)^{\alpha-2}]$$

$$= \delta A\alpha(\alpha-1)[\beta^{\alpha-2} + (1-\beta)^{\alpha-2}]$$

< 0, as $\alpha < 1$

Hence W(β) is concave in β.

Optimality condition for β: Setting the first order condition for maximization of W(β),

$$\frac{dW}{d\beta} = 0 \Rightarrow \delta A[\alpha\beta^{\alpha-1} - \alpha(1-\beta)^{\alpha-1}] - c = 0 \Rightarrow \delta A\alpha[\beta^{\alpha-1} - (1-\beta)^{\alpha-1}] = c$$

$$\Rightarrow \beta^{\alpha-1} - (1-\beta)^{\alpha-1} = \frac{c}{A\delta\alpha} \text{---------} (**)$$

The welfare optimizing β is derived from the above condition.

APPENDIX F

Taking total differentiation in optimal condition (**) of Appendix E,

$$[(\alpha-1)\beta^{\alpha-2} + (\alpha-1)(1-\beta)^{\alpha-2}]d\beta = \frac{1}{A\delta\alpha}dc - \frac{c}{A\alpha\delta^2}d\delta - \frac{c}{\alpha\delta A^2}dA$$

Taking $d\delta = 0,\ dA = 0, (\alpha-1) < 0$

$$\frac{d\beta}{dc} = \frac{1}{A\delta\alpha[(\alpha-1)(\beta^{\alpha-2}+(1-\beta)^{\alpha-2})]} < 0$$

Taking $dc = 0,\ dA = 0, (\alpha-1) < 0$

$$\frac{d\beta}{d\delta} = -\frac{c}{A\alpha\delta^2[(\alpha-1)(\beta^{\alpha-2}+(1-\beta)^{\alpha-2})]} > 0$$

Taking $d\delta = 0,\ dc = 0, (\alpha-1) < 0$

$$\frac{d\beta}{dA} = -\frac{c}{A^2\alpha\delta[(\alpha-1)(\beta^{\alpha-2}+(1-\beta)^{\alpha-2})]} > 0$$

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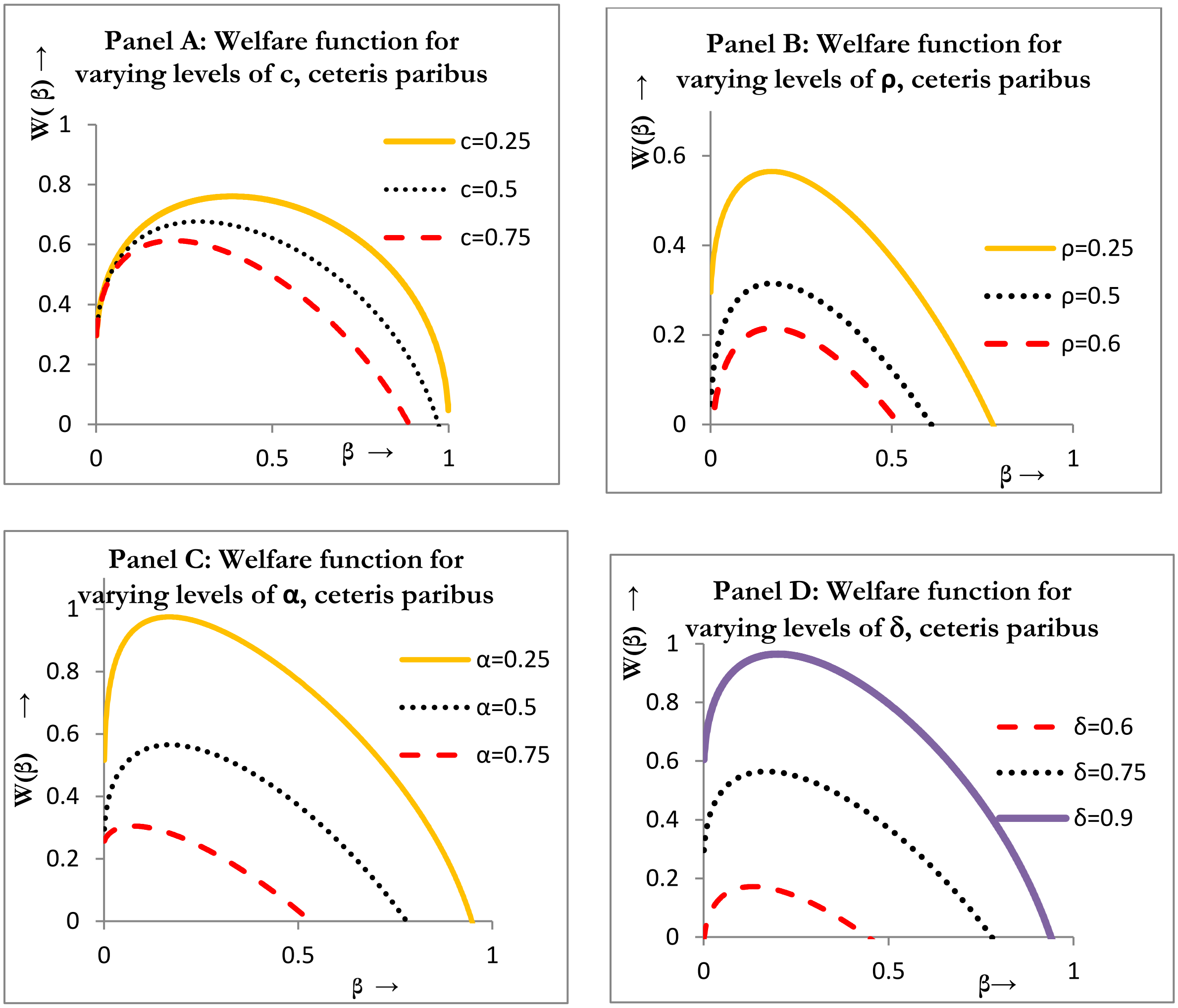

FIGURE 1

An Illustration of the Welfare Function W(β)

This figure illustrates the concave nature of the welfare function in credit quota parameter β. Plots of the welfare function simulated for various hypothesized values of the set of parameters (α, ρ, δ, c, A) are presented in various panels. In Panel A, we keep (α=0.25, ρ=0.25,δ=0.75, A=2) fixed and compute the welfare function corresponding to three different values of c as depicted in the graph. In Panel B, computed welfare functions for three different values of the parameter ρ are shown, while keeping (α=0.5, c=1, δ=0.75, A=2) fixed; Panel C and Panel D similarly present respectively the plots of the welfare functions computed for changing α for fixed values of (c=1, ρ=0.25, δ=0.75, A=2) and that computed for changing levels of δ with fixed levels of the other parameters (α=0.5, ρ=0.25, c=1, A=2).

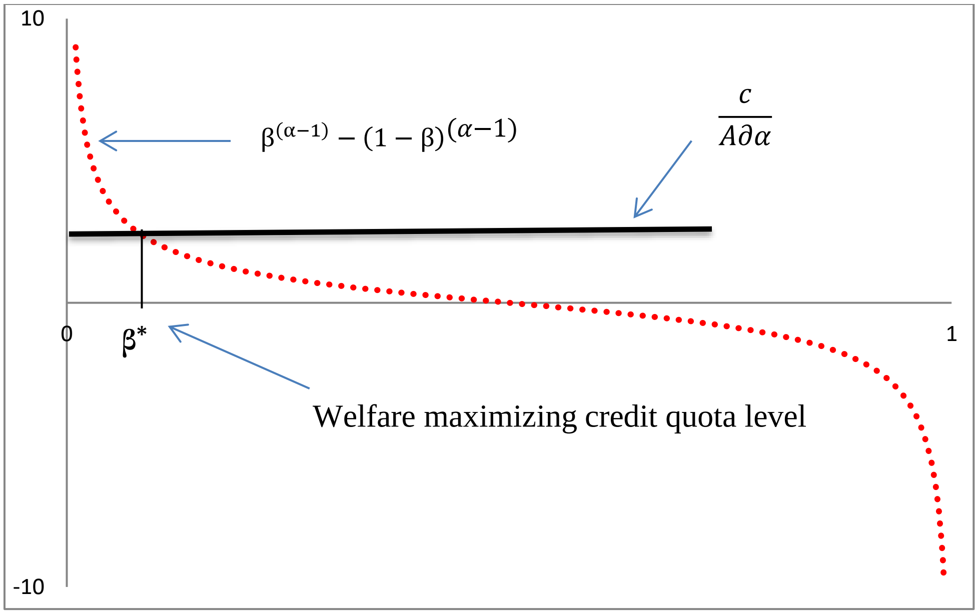

FIGURE 2

Illustration of Optimal Credit Quota

The dotted graph in Figure II represents the left side of equation (6) while the horizontal solid line represents the right side of it. The point of intersection gives the optimal level of credit quota.