Discussion Papers in Economics

Political Competition, Party Structure and Economic Growth: Theory and Evidence from Indian States

J. Stephen Ferris and Bharatee Bhusana Dash

Discussion Paper 25-02

Centre for International Trade and Development

School of International Studies

Jawaharlal Nehru University

India

*

Political Competition, Party Structure and Economic Growth: Theory and Evidence from Indian States[†]

J. Stephen Ferris[‡]
Department of Economics, Carleton University, Canada
(SteveFerris@cunet.carleton.ca)

and

Bharatee Bhusana Dash
Center for International Trade and Development, Jawaharlal Nehru University, India
(bharatee.dash@gmail.com)

Abstract

This paper uses a two period overlapping generations model with balanced growth to investigate the links arising among political competition, the effective number of political parties (ENP), the composition of government spending and the growth rate of the economy. The model highlights three hypotheses with respect to political competition and ENP. First, while a small rise in ENP is required to breakdown oligopolistic political power, a further rise will fragment the credibility of opposition to the incumbent governing party, lessening effective competition and leading to operational inefficiency and excessive government size. The second hypothesis argues that an increase in party competitiveness requires a compositional output response leading to a more consumption intensive package of government services. Third, effective party competition is complementary with economic growth. All three imply a non-monotonic relationship with ENP. A panel of annual data on 14 major Indian states spread over six decades is used to test these predictions and the results suggest that the data from Indian states fit well with the predictions of the model.

JEL Categories: H0, H5, D7, D9, P16
Key Descriptors: Political Competition, political party structure, government size, economic growth, overlapping generations

---

[†] This paper extends work begun with Stanley Winer and Bernie Grofman on political competition and its application to India and Canada. We acknowledge the helpful comments and suggestions of Pintu Parui and the participants of 19th Annual Conference on Economic Growth and Development at ISI, Delhi, and CITD seminar series, JNU, Delhi. The usual disclaimer applies.
[‡] Corresponding author.

1. Introduction

The Duverger-Demsetz hypothesis argues that the first-past-the-post characteristic of majoritarian voting leads to electoral convergence on two political parties. In this paper we extend Duverger-Demsetz competition (Duverger 1954; Demsetz 1968, 2008) by focussing on characteristics of political party competition and the effective number of political parties (ENP).[1] To do so we follow Carmec et. al. (2019) and use a two period overlapping generations model with balanced growth to link party competition, ENP, the composition of government services and the growth rate of the economy. The model highlights three hypotheses with respect to political competition and ENP. First, while effective competition requires some minimum number of electorally credible parties, a rise in the effective number of parties above that minimum fragments the credibility of opposition to the incumbent governing party, lessening effective competition, leading to operational inefficiency and resulting in excessive government size. Hence while the relationship between party competitiveness and ENP has an inverted U-shaped, the implied relationship between ENP and government size will be U-shaped. The second hypothesis argues that as ENP increases, the range of party platforms offered electorally also increases. That is, as party competition increases a winning electoral strategy will require the promise and delivery of a broader range of government services to span the wider set of options proposed by competitors. Moreover, the immediacy of electoral competition encourages the incumbent governing party to substitute platforms featuring consumption services (Chhibber and Kollman, 1998; Baraldi, 2008; Lewis and Hendrawan, 2019; and Scartascini and Crain, 2021). However, as ENP continues to rise and fragmentation leads to a reduction in effective party competition, the need to match rival offers diminishes and the composition of government services becomes less consumption intensive. Finally, an increase in effective party competition is hypothesized to be associated not only with a smaller government size but also with a greater investment mix of government services, reducing inefficiency and

[1] ENP is defined as $ENP_{jt} = 1/\sum_{i=1}^{I} s_{ijt}^{2}$ where s_i is the seat (or vote) share of party i in the legislature of state j at time t.

resulting in a higher rate of growth. In all three cases the relationship with ENP is expected to be non-monotonic.[2]

In the following section we present a growth model that incorporates these features. The second part of the paper tests the hypotheses generated on a panel of data from 14 large Indian states encompassing over 85 percent of the Indian population and covering the period from 1959/60 to 2019/20.[3] India's states provide a useful case study of political party competition, government size and economic growth because the size and heterogeneity of India's population has resulted in a wide variety of political party structures.[4] For example, it is not unusual for an Indian state election to feature more than 100 parties. Even when weighting parties by the percentage of the seats won, India stands out in comparison to other Westminster parliamentary democracies as featuring a larger effective number of competing parties in both their center and state governments.[5]

The paper is structured as follows. In section 2 we present a formal model studying the intertwining relationships among political party structure, fiscal responses and economic growth in a majoritarian democracy. Section 3 presents the data and empirical strategy, while section 4 discusses findings of the paper. Section 5 concludes. An Appendix provides discussion on data, and derivations and proofs of the model.

[2] See Durham (1999) and Acemoglu and Robinson (2006) for analyses that posit a non-monotonic relationship between political competition (viewed as a spectrum running from autocracy through democracies) and the rate of economic growth. This analysis considers the degree of political competition as reflected in one government institution of contemporary democracies—the effective number of political parties. See Ferris and Voia (2023) and Ferris and Dash (2024).

[3] The 14 Indian states included in our study are: Andhra Pradesh, Bihar, Gujarat, Haryana, Karnataka, Kerala, Madhya Pradesh, Maharashtra, Odisha, Punjab, Rajasthan, Tamil Nadu, Uttar Pradesh, and West Bengal. Assam was excluded because it was subdivided twice during the 70's and 80's and because it has experienced long periods of communal tension with associated outbreaks of violence. Jammu and Kashmir was excluded for a similar reason.

[4] See Ferris and Dash (2023) for a discussion of the factors determining party structure across Indian states.

[5] Chhibber and Kollman (2004, Tables 1.1 to 1.4) note that over the 1960 to 2000 time period the effective number of parties in India (between 4 and 7) was much higher than in Canada, Britain or the U.S. (2 to 4). Dash et al. (2019) document the average effective number of parties in Indian states as between 3 and 6 over the period 1952 to 2009.

2. A model of the role of political party structure on government investment, size and economic growth in majoritarian democracies

A. Representative firm behaviour and the output decision.

At time t a representative firm, i, is assumed to produce a composite output, Y_{it}. Using Cobb-Douglas technology and assuming that the government finances its spending through a unit tax levied on output, the private income/output generated by the firm is

$$Y_{it} = (1-\tau)\theta K_{it}^{\alpha} L_{it}^{1-\alpha} K_{g_t}^{\omega} = (1-\tau)\theta L_{it}\, k_{it}^{\alpha} K_{g_t}^{\omega} \qquad \text{where } 0 < \alpha, \omega < 1 \qquad (1)$$

Where θ is the level of technology, K_{it} is private capital, K_{g_t} is a public good, capital provided by the government, L_{it} is labour used by the firm, τ is the unit tax levied by government on output and $k_{it} = \frac{K_{it}}{L_{it}}$. The factor share of private capital in production is $0 < \alpha < 1$ and $0 < \omega < 1$ is the share of publicly provided capital. Output per worker can then be written as $y_{it} = (1 - \tau)\theta\, k_{it}^{\alpha} K_{g_t}^{\omega}$.

If capital depreciates entirely after use and K_{g_t} is parametric to the firm, the firm will maximize profits by increasing its use of capital and labour until,

$$\frac{\partial Y_{it}}{\partial K_{it}} = (1-\tau)\theta\alpha k_{it}^{\alpha-1} K_{g_t}^{\omega} = R_t, \text{ where } R_t \text{ is the cost of capital to the firm.} \qquad (2)$$

$$\frac{\partial Y_{it}}{\partial L_{it}} = (1-\tau)\theta(1-\alpha) k_{it}^{\alpha} K_{g_t}^{\omega} = w_{it}, \text{ where } w_t \text{ is the wage paid to each worker.} \qquad (3)$$

As all i firms are identical, all firms choose the same capital-labour ratio so that the after tax output produced per worker, can be expressed as

$$\frac{Y_t}{L_t} = y_t = (1-\tau)\theta\left(\frac{K_t}{L_t}\right)^{\alpha} K_{g_t}^{\omega} = (1-\tau)\theta k_t^{\alpha} K_{g_t}^{\omega}, \qquad (4)$$

where $k_t = \frac{K_t}{L_t}$, $L_t = \sum L_{it}$ and $L_t = L$. With K_{g_t} determined by the government sector and the aggregate size of the private capital stock predetermined by the savings decision of older households made in the previous period, the competitive rate of return on capital is

$$R_t = (1-\tau)\alpha\theta k_t^{\alpha-1} K_{g_t}^{\omega} = \alpha(1-\tau)\frac{y_t}{k_t}. \qquad (5)$$

Similarly with a competitive labour market, the wage received by private and government workers is,

$$w_t = (1-\tau)(1-\alpha)\theta k_t^{\alpha} K_{g_t}^{\omega} = (1-\tau)(1-\alpha)y_t. \quad (6)$$

Note that R_t is linear in the output/capital ratio, $\frac{y_t}{k_t}$, while the wage is linear in per capita output, y_t. It follows that if the tax rate stays constant, an increase in the supply of the public good provided by the government, K_g, will increase output per worker at a decreasing rate as well as increase the return to private capital and the wage rate received by all workers.[6] An increase in the tax rate will decrease output per worker and the after tax returns to capital and labour.

B. The household's consumption decision

We assume that all individuals live for two periods. Work takes place only in the first period so that individuals must save, x_t, to spread consumption over their lifespan. Savings takes the form of capital which will be used for production in the second period. Government consumption services are assumed to be private goods and provided equally to consumers only when individuals are young (e.g., receive early health and education services) and all savings are used for private consumption in the retirement period of life. Individuals are assumed to have the same logarithmic utility functions, $U(c_1) = ln\, c_1 + \gamma\, ln\, g(N)$ and $U(c_2) = ln\, c_2$ and have a common rate of time preference, $0 < \beta < 1$. Taxes are imposed on the producer so that wage and rate of return are after tax rates.

With $g_t(N_t)$ and τ determined by the political process, and treating the prices as parameters, the individual household's choice problem is to choose $c_{1,t}$ and $c_{2,t+1}$ to

$$Max\; ln\, c_{1t} + \gamma\, ln\, g_t(N_t) + \beta\, ln\, c_{2,t+1} \text{ subject to } c_{1,t} = w_t - x_t;\; c_{2,t+1} = R_{t+1}x_t \quad (7)$$

[6] That is, $\frac{\partial y_t}{\partial K_{g_t}} = \omega(1-\tau)\theta k_t^{\alpha} K_{g_t}^{\omega-1} > 0$, $\frac{\partial^2 y_t}{\partial K_{g_t}^2} = \omega(\omega-1)(1-\tau)\theta k_t^{\alpha} K_{g_t}^{\omega-2} < 0$, $\frac{\partial R_t}{\partial K_{g_t}} = \omega\alpha(1-\tau)\theta k_t^{\alpha-1} K_{gt}^{\omega-1} > 0$, and $\frac{\partial w_t}{\partial \omega} = \omega(1-\alpha)(1-\tau)\theta k_t^{\alpha} K_{g_t}^{\omega-1} > 0$.

with publicly provided private good $g_t(N_t)$ given and where savings $= x_t = k_{t+1}$ for each household[7]. Substituting the constraints into the objective function, the problem becomes

$$Max\ L(x_t) = ln\,(w_t - x_t) + \gamma\, ln\, g_t(N_t) + \ \beta\, ln\,\big((1 + r_{t+1})x_t\big) \tag{8}$$

The first order condition for an optimal choice is $\frac{\partial L}{\partial x_t} = \frac{-1}{w_t - x_t} + \beta\frac{(1+r_{t+1})}{(1+r_{t+1})x_t} = 0$, which simplifies to $x_t = \frac{\beta}{(1+\beta)}w_t$. Substituting $w_t = (1-\tau)(1-\alpha)y_t$ from (6),

$$x_t = k_{t+1} = \frac{\beta(1-\tau)(1-\alpha)}{(1+\beta)}y_t. \tag{9}$$

From this it follows that the rate of growth in the economy is

$$\frac{k_{t+1}}{k_t} = \ \frac{\beta(1-\tau)(1-\alpha)}{(1+\beta)}\left(\frac{y_t}{k_t}\right). \tag{10}$$

Since private output per capita in equation (4) is a linear function of per capita private capital, the model resembles the classic *AK* model with the linear difference equation of per capita capital in equation (10) becoming the balanced growth rate of the economy. Using the budget constraint and (6) to solve for the optimal consumption choices, we find

$$c_{1,t} = \ w_t - \frac{\beta}{(1+\beta)}w_t = \ \frac{1}{(1+\beta)}w_t = \frac{(1-\tau)(1-\alpha)}{1+\beta}y_t \tag{11}$$

Note that with the tax rate fixed, period 1 consumption is a linear function of per capita income, y_t. Under balanced growth $\frac{y}{k}$ is constant over time so that period 2 consumption will also be a linear function of y_t. That is, substituting for R_t from (5) we find

$$c_{2,t+1} = R_{t+1}\frac{\beta(1-\tau)(1-\alpha)}{(1+\beta)}y_t \ = \frac{\alpha\,\beta(1-\tau)^2(1-\alpha)}{(1+\beta)}\left(\frac{y_{t+1}}{k_{t+1}}\right)y_t. \tag{12}$$

C. The government budget constraint

Political parties compete for votes by offering two types of government services, $g_t(N_t)$ to households, and K_{g_t} to firms. These are paid for by a unit tax on all income, τ. We assume that

[7] Because individuals save only when they are young at time t, x_t, savings takes the form of the capital private firms use in next period $t+1$, k_{t+1} (see Barro and Sala-i-Martin (2004), Ch. 3).

the government budget must be balanced so that total government expenditure must be covered by tax revenue. That is,

$$T_t = \tau Y_t = L * g_t(N_t) + K_{g_t} + Z(N_t), \quad (13)$$

where $Z(N_t)$ is the cost of a political system with N_t effective parties. $Z(N_t)$ is viewed as the agency cost of government which depends upon the degree of political competition and where effective competition is a function of the effective number of parties, N_t. Rewriting the budget constraint in per private sector worker terms we find,

$$\tau y_t = g_t(N_t) + \frac{K_{g_t}}{L} + z(N_t), \text{ where } z(N_t) = \frac{Z(N_t)}{L}. \quad (14)$$

An increase in N_t increases directly $g_t(N_t)$ while $z(N_t)$ will be modelled as having a U-shaped effect on government expenditure.

D. Political Parties and Competition

Political parties compete to govern by proposing a policy platform that consists of a level of government services split between a public good, K_{g_t}, and consumption services, $g_t(N_t)$, and set a tax rate, τ, such that the government budget constraint is met. We assume that there are private party benefits derived from being the governing party, $z(N_t)$, and these agency costs of providing government services are assumed to be controlled by the degree of political competition which in turn is a non-monotonic function of N_t.[8] We assume that the effective number of competing political parties, N_t, is determined by the costs and benefits of party participation (political institutions and customs determined outside of the model). In any given political environment, both N_t and $z(N_t)$ are constants. However, the effect of an increase in N_t on agency costs will depend upon whether or not N is greater or smaller than N_{min}, the

---

[8] Recent research suggests that $z(N)$ has a U-shape, first falling as a larger number of effective parties offsets the joint incentive that monopolistically competitive parties have to collude at the expense of the electorate. However, as entry continues, the winner-take-all nature of a majoritarian election means that more parties will decrease the likelihood that any one challenger will be a credible rival to the incumbent. That is, above some minimum, N_{min}, further entry fragments the vote among parties reducing the credibility of parties as effective monitors of the behaviour of the governing party. See Ferris, Winer and Grofman (2016), Ferris and Voia (2023) and Ferris and Dash (2024).

number of effective competitors that generates the degree of competitiveness that minimizes agency costs. That is, $z(N = 1) > 0$ and $z_N < (>) 0$ and $z_{NN} > (<) 0$ when $N < (>) N_{min}$.[9]

The representative household prefers the party that offers a wider range of consumption services. Because competing parties offer both overlapping and distinctive services the winning party is led to offer a wider range of consumption services as the effective number of competing political parties increases.[10] Hence as N_t increases, $g_t(N_t)$ increases but at a decreasing rate, i.e., $g_N > 0$ and $g_{NN} < 0$.

More formally, the winning party is assumed to maximize a political support function that is based on the utility received by current voters. Using $W(U(c_{1t}, c_{2t}, g_t(N_t))$ to represent the political support function, the appropriate strategy for the winning party is to $Max\, W(g_t(N_t), K_{g_t}, \tau_t)$ subject to the budget constraint, $\tau y_t = g_t(N_t) + \frac{K_{g_t}}{L} + z(N_t)$. Using the Lagrangian,

$$W\big(g_t(N_t), K_{g_t}, \tau\big) = ln\, c_{1t} + ln\, c_{2t} + \gamma\, ln\,(g_t(N_t)) + \mu\left\{\tau y_t - g_t(N_t) - \frac{K_{g_t}}{L} - z(N_t)\right\}. \quad (15)$$

where from (11) $c_{1t} = \frac{(1-\tau)(1-\alpha)}{1+\beta} y_t$ and from (5) $c_{2t} = R_t k_t = \alpha(1-\tau) y_t$ (since k_t is determined by the savings decisions made by the current old in the previous period).

The first order conditions for that maximize support for the winning political party are[11]

$$\frac{\partial W}{\partial g_t(N_t)} = \frac{\gamma}{g_t(N_t)} - \mu = 0 \quad (16)$$

$$\frac{\partial W}{\partial K_{g_t}} = (1-\tau)\omega\theta k_t^{\alpha} K_{g_t}^{\omega-1} - \frac{1}{L} = 0 \quad (17)$$

---

[9] We follow the convention of representing the first derivative of z with respect to N as z_N, and the second derivative as z_{NN}.

[10] Particularly in a first-past-the-post voting system each party needs lesser number of votes to win an election as the number of competing parties increases at the equilibrium. Under such circumstances, targeting the core voters by delivering them relatively private goods, largely budgeted as public consumption spending, becomes an effective electoral strategy for the competing parties. See Chhibber and Nooruddin (2004), Dash and Raja (2013) and Winer et al (2021) for related works in an Indian context.

[11] We assume that because information is limited the government views its provision of capital as affecting only firm output and through this its tax revenues and, second, that an increase in its tax rate will raise its revenues at the cost of lowering household incomes (and so electoral support). Second order follow on effects are not incorporated in government decision making (but do influence the general equilibrium solution).

$$\frac{\partial W}{\partial \tau_t} = -\frac{2}{(1-\tau)} + \mu y_t = 0 \tag{18}$$

$$\frac{\partial W}{\partial \mu} = \tau y_t - g_t(N_t) - \frac{Kg_t}{L} - z(N_t) = 0 \tag{19}$$

The first three first order conditions capture the optimal political trade-offs arising under the winning electoral strategy. From (17) it can be seen that optimal investment behaviour on the part of the government is to increase the quantity of public capital as long as its marginal contribution to private output, $L * (1-\tau)\omega\theta(1-\lambda)k_t^{\alpha}K_{g_t}^{\omega-1}$, exceeds its cost in terms of foregone government consumption (i.e., 1). Equation (16) tells us that government consumption services should be increased as long as its marginal value to households (which is falling in g_t) is greater than what is lost by having to increase the government's budget. Equation (18) tells us that the cost of having larger government consumption and investment is having to raise taxes which in turn reduces household income and hence households' consumption of private out. That is, putting (16) and (18) together the optimal behaviour implies, $\frac{\gamma}{g_t(N_t)} = \frac{2}{(1-\tau)y_t}$. The final equation (19) keeps the government's alternatives constrained within its budget. Given diminishing marginal utilities and the diminishing marginal product of capital (imposed by the assumed forms of the utility and production functions), the conditions in (16) through (19) are sufficient to solve for the optimal values $g_t(N_t)^*$, $K_{g_t}^*$ and τ^*.

E. The effect of an increase in the effective number of political parties

A change in the political environment of an economy that allows for the entry of new parties and results in an increase in the effective number of parties will perturb the political equilibrium described above and so affect the composition of the government's output and the rate of growth of the economy.[12] Factors that can lead to an exogenous change in the number of parties or produce changes across different political jurisdictions include differences in the degree of public funding/subsidization of political parties and/or differences in institutional requirements such as in the minimum vote requirements for party status in the legislature (see

[12] Factors that can lead to an exogenous (to our model) change in the number of parties include the public funding/subsidization of political parties and/or changes in minimum vote requirements for representation in a legislature (in India's case, see Ferris and Dash (2023)). Similarly a change in the voting rules, such as a change from majoritarian to proportional voting, can result in a change in the effective number of parties.

Mendilow (1992) for Israel and Ferris and Dash (2023) for India). Similarly changes in a voting rule, such as a change from a majoritarian to proportional voting system, can result in a change in the effective number of parties. In our model the effect of a change in N for one of these reasons can be solved for by totally differentiating the political equilibrium represented by equations (16) through (19) with respect to N and solving for the corresponding changes in the endogenous variables. The mathematics of this procedure is presented in the appendix to this paper, here we present only the final outcomes and intuition lying in that solution.

As we have modelled party competition, an increase in N_t increases the range of government consumption services offered and this in turn requires an increase in the tax rate and/or a decrease in government capital services to preserve budget equilibrium. In general, both of these will change, adversely affecting both final output and household income. Because our focus is specifically on the growth rate, the effect of a change in N can be seen directly from (10), where $\frac{k_{t+1}}{k_t} = \frac{\beta(1-\tau)(1-\alpha)}{(1+\beta)}\left(\frac{y_t}{k_t}\right)$. Differentiating the growth rate with respect to N, we see that

$$\frac{d(\frac{k_{t+1}}{k_t})}{dN} = -\frac{\beta(1-\alpha)}{(1+\beta)}\left(\frac{y_t}{k_t}\right)\left(\frac{d\tau}{dN}\right) + \frac{\beta(1-\tau)(1-\alpha)}{(1+\beta)}\left[-\theta k_t^{\alpha-1}K_{g_t}^{\omega}\left(\frac{d\tau}{dN}\right) + \omega(1-\tau)k_t^{\alpha-1}K_{g_t}^{\omega-1}\left(\frac{dK_{g_t}}{dN}\right)\right] \quad (20)$$

depends upon the sign and size of the two general equilibrium effects $\left(\frac{d\tau}{dN}\right)$ and $\left(\frac{dK_{g_t}}{dN}\right)$. From the appendix, the two total derivatives, $\left(\frac{d\tau}{dN}\right)$ and $\left(\frac{dK_{g_t}}{dN}\right)$ can be signed. Because both derivatives depend upon z_N, when $N < N_{min}$ so that an increase in party competition helps to break down political market power, $\left(\frac{d\tau}{dN} < 0\right)$ and $\left(\frac{dK_{g_t}}{dN} > 0\right)$. Hence the increase in competitiveness leads to an increase in the growth rate. In the case where $N > N_{min}$ political competition becomes increasingly excessive so that $z_N > 0$ and $\left(\frac{d\tau}{dN} > 0\right)$ and $\left(\frac{dK_{g_t}}{dN} < 0\right)$. Here an increase in N reduces the growth rate.

It follows that the effect of a change in the political environment that leads to a change in the effective number of political parties is predicted to have a non-monotonic effect on government size, the tax rate, the mix of government consumption and investment services and the growth rate. In the following section we test for the predicted effects.

3. Empirical Implementation

A. Testing strategy

Measuring the size of agency costs in the public budget is notoriously difficult. In our empirical analysis, we assume that changes in agency costs are reflected in complementary changes in the aggregate expenditure size of the public sector as predicted by our theory (in response to changes in political party competitiveness as measured by ENP). Using this metric we test three predictions that follow directly from the above model. First, an increase in ENP has a U-shaped effect on the average unit tax or government size. This follows from the assumption that ENP is a non-monotonic measure of the degree of effective political competition, first representing a rise in political competitiveness by reducing the degree of oligopolistic political market power before further increases reduce the effectiveness of party competition by producing electoral fragmentation.[13] Second, growing party competitiveness requires the governing party to respond to rival party platforms by broadening the range change of contemporary government services and thus producing a wider range of consumption services. Third, growing party competitiveness not only reduces government size and lowers taxes but also results in a more efficient government producing higher quality policies. Together these imply an increase in the growth rate and an inverted U-shaped relationship with ENP. Because effective party competition is non-monotonically related to ENP, all three hypothesized relationships are expected to be non-monotonic.

Our test has three stages. The first stage extends Ferris and Dash (2024) who used election year data from Indian states to test the hypothesis that political competition (as measured by ENP) and government size have a U-shaped relationship with the minimum point representing the level of ENP that minimizes government agency costs through party competition. Here we use annual data on two measures of state government size: the share of non-interest government

[13] In a majoritarian political system, the winner-take-all nature of electoral competition means that a larger effective number of parties will fragment the likelihood that any single competitor will be a credible rival thus lowering contestability and the level of effective political competition. In earlier work we have referred to this as Duverger-Demsetz competition. On the other hand, as the effective number of parties falls towards 2, collusion can arise among the smaller number of effective rivals allowing them to promote party specific goals at the expense of the electorate. See Ferris, Winer and Grofman (2016), Ferris and Voia (2023) and Ferris and Dash (2024).

expenditure in state gross domestic product (Govsize) and the revenue size of government or $\frac{T}{Y}$ (the unit Tax rate). Recognizing that these two measures of state government size, ENP(Seat), real income per capita (Rypc), and our other control variables have different degrees of stationary, the analysis uses the panel error correction form of an autoregressive distributed lag (ARDL) model to separate long and short run influences on our measures of government size.

The general form of the panel ARDL model used for our sample of Indian states is

$$G_{i,t} = \beta_0 + \sum_{\boldsymbol{j=1}}^{\boldsymbol{j=2}} \beta_j ENP(Seat)_{i,t}^{j} + \beta_3 G_{i,t-1} + \beta_4 X_{i,t-1} + \mu_i + \varepsilon_{i,t} \tag{21}$$

where G is a fiscal variable for state i at time t, $ENP(Seat)$ is our variable of interest with $j = 1, 2$ representing linear and quadratic terms. Here a negative/positive sequencing of coefficients would confirm a U-shaped relationship. X represents the set of other explanatory variables that may play a pivotal role in determining the fiscal outcome variable. μ is a state-specific fixed-effect accounting for state-specific, time invariant effects. ε is the random error term.

If all variables in (21) are I(0) or I(1) and cointegrated, then the error term is an I(0) process for all i. The attractive feature of cointegrated variables is their responsiveness to any deviation from long run equilibrium. This feature leads to an error correction process where the short run dynamics influence the deviation from equilibrium. By re-parameterizing equation (21), we can estimate the long run relationship between variables using the error correction equation

$$\Delta G_{i,t} = \phi_i \left[G_{i,t-1} - \left(\alpha_0 + \sum_{j=1}^{j=2} \alpha_j ENP(Seat)_{i,t}^{j} + \alpha_3 X_{i,t} \right) \right] + \gamma_1 \Delta G_{i,t-1} + \gamma_2 \, \Delta X_{i,t-1} + \mu_i + e_{i,t} \tag{22}$$

where the expression inside the square bracket estimates the long run relationship between variables and ϕ_i is the error correction term that determines the speed of convergence to the long run equilibrium. The error correction coefficient must lie strictly between 0 and 1 to confirm a stable long run relationship.

Following Pesaran and Smith (1995) and Pesaran, Shin and Smith (1999), the error correction modelling allows three options regarding the treatment of the model's long and short run coefficients: pooled mean group modelling, *pmg*, where the long run coefficients are constrained to be equal across states; mean group modelling, *mg*, where the coefficients are calculated as the unweighted average of the unconstrained model; and dynamic fixed effects, *dfe*, where all parameters are constrained to be equal across states. A Hausman specification test is then used to indicate the more efficient estimation method.[14]

Investigating the non-monotonic relationship between the ENP(Seat) and fiscal outcome variable through equations (21) and (22) imposes a symmetric quadratic restriction on the data. This restriction will bias the shape if the true shape is asymmetric about the extreme point. For a better exposition of the quadratic relationship, we follow Leonida et al (2013, 2015) and test for non-monotonicity using a fixed-effects version of fractional polynomial (fp) analysis.[15] The *fp* procedure in Stata uses 44 combinations of the powers of $k = (-2, -1, -.5, 0, .5, 1, 2, 3)$ to find the best fitting second degree fractional polynomial of fiscal outcome variable within a regression of ENPs on our control variables

$$G_{i,t} = \alpha_0 + \sum_{\boldsymbol{j=1}}^{\boldsymbol{j=2}} \alpha_j ENP(Seat)_{i,t}^{k} + \alpha_3 X_{i,t} + \mu_i + \epsilon_{i,t} \quad (23)$$

The second stage tests for the hypothesized inverted U-shaped relationship between the ratio of government consumption to investment spending and ENPSeats. The same estimation strategy used in the first stage is used in the second stage tests. A positive linear ENP(Seat) coefficient and a negative quadratic coefficient would confirm an inverted U-shaped relationship.

The third stage tests the overall hypothesis that the rate of growth of per capita income and ENP(Seat) has an inverted U-shaped relationship. Because the growth rate is a stationary variable (as confirmed by the panel unit toot test presented in the appendix), we estimate using a standard dynamic fixed-effects model where all included variables are in their stationary form

$$\Delta Y_{i,t} = \alpha_0 + \sum_{j=1}^{j=2} \alpha_j ENP(Seat)_{i,t}^{j} + \alpha_3 \Delta Y_{i,t-1} + \alpha_4 X_{i,t} + \mu_i + \epsilon_{i,t} \quad (24)$$

---

[14] The command *xtpmg,* implemented in Stata by Blackburne III and Frank (2007), is used to estimate all three versions of ARDL specification and select the efficient estimation method.

[15] The *fp* procedure allows for a wide range of shapes that allows for the determination of the best fitting flexible form without predetermining its shape. We also checked for a broader range of non-monotonic relationships by increasing the dimensions up to m=4 to confirm that 2 dimensions (m=2) and an asymmetric quadratic shape was still the best fit. For this reason only the m=2 results and/or shapes are presented for our tests.

where ΔY is the growth rate of per capita income. Since our model predicts an inverted U-shaped relationship between per capita income growth and ENP(Seat), we expect the positive/negative sequencing of the coefficients of linear and quadratic ENP(Seat) terms. To again investigate the possibility of an asymmetric quadratic relationship between per capita income growth and ENP(Seat), we use a fixed-effects version of the *fp* model shown in equation (23) with per capita income growth rate as the outcome variable.

In all cases we look for the models predicting a peak in party competitiveness at an ENP value that is neither below Duverger's 2 nor too far above. Confirmation would then provide evidence consistent with the hypothesis that ENP is an effective non-monotonic metric of the degree of political party competition and the positive role of party competition in relation to economic growth.

B. Data and variables used in the tests

The sources of the panel data used to test the model's predictions are provided in the Data Appendix to the paper along with their descriptive statistics. The statistics themselves indicate considerable variation across Indian states. For example, in 2019/20 real per capita income in Karnataka was more than four and a half times larger than its counterpart in Bihar and similar variability is present in literacy rates, urbanization and the output share of government investment across states. Variation arises not only across states but over time with differences in the time series properties of variables playing an important role in our tests. Except for the percentage of the population that is older than sixty (Old), all variables are either I(1), growing stochastically over time, or I(0), stationary through time.[16] This is the feature that motivates the use of autoregressive distributed lag regressions in the first two stages of our test.

The dependent variables in the first stage of our test are: government expenditure size (GovSize), defined as the ratio of aggregate noninterest government expenditure to state GDP, and government revenue size (Tax rate), the ratio of total state government revenues to state GDP. The government consumption to investment ratio (GCons_mix) is defined as ratio of state

[16] Regressing nonstationary variables raises the possibility of estimated relationships being spurious. In our dataset the variable Old is I(2), increasing stochastically at an increasing rate. Hence Old appears as either a first or second difference in our tests. Panel unit root results based on Fisher Test are available in descriptive statistics table.

government revenue to capital expenditures. To minimize stationarity issues these variables are transformed into logarithms in our tests. Economic growth is defined as the growth rate of state real income per capita (Grypc). Our primary variable of interest, ENPSeat is defined as one over the sum of each party's seat share in the state legislature. The control variables used to account for heterogeneity across the factors and with the potential to influence the four dependent variables include: real state income per capita (Rypc), the percentage of the population that is literate (literacy), the percentage of the population older than 60 (Old), the share of agriculture in state GDP (Agriculture_share), the percentage of the state population living in urban areas (Urban); the average population size of state electoral constituencies (in 1000s, Density); the percentage of seats reserved for disadvantaged groups (Reservation); the fraction of state expenditures financed by intergovernmental grants (Grant_share), whether or not the state has a fiscal rule restraining the size of the state budget deficit (Fiscal_rule = 1 if a rule is present, 0 otherwise), and a variable indicating the use a value added tax (VATdummy = 1 when the state had a value added tax, 0 otherwise).[17] While our dataset is annual, not all variables are available annually. Periodic variables were annualized by interpolating data between years. Finally, different control variables appear in different tests as appropriate. For example, while all models include a core mix of socioeconomic variables to capture the fundamentals of the economy, variables such as Grant_share, Fiscal_rule and VATdummy are included only in models where these variables are likely to affect size, consumption mix or growth rate directly.

4. Empirical Results

Stage 1: The relationship between ENP(Seat) and both GovSize and Tax_rate

Applying the ARDL model specified in equation (22), the results for the three versions of the error correction form of government size (Govsize) are shown in Table 1. Column (1) presents the dynamic fixed effects (dfe) version of that model, where the *dfe* algorithm imposes

[17] At the state level the major (indirect) tax reform introduced in our period was the shift from an inefficient sales tax regime to a value added tax (VAT) which resulted in a discrete increase in state government revenues. The states in our sample adopted the VAT in the years between 2003 and 2008. Similarly, to address the deteriorating budget deficit situation, Indian Parliament enacted Fiscal Responsibility and Budget Management Act (FRBMA) in 2003. The adoption of Fiscal rule took place across states between 2003 and 2010.

common covariant coefficients across states in both the long and the short run. Column (2) assumes a common long run but allows for variation across states in the short run by presenting the pooled mean group (pmg) coefficient values. Column (3) presents coefficients generated as the unweighted group average (mg) of the unconstrained model. Inspection of Table 1 reveals that the coefficient estimates across all models are broadly similar in sign and size. In comparative terms, however, a Hausman specification test indicates the *pmg* model in column (2) (that assumes a common long run across states) is the preferred model. To account for the temporal feature of the data, we present in column (4) an extended *pmg* model that includes a time trend. The restrictions the *pmg* model imposes on data also fit well with the institutional features of the Indian federation where individual states have some freedom to deviate away from federally issued fiscal mandate in short run but all states are guided by the same set of federal rules in long run.

-- insert Table 1 about here --

For our purposes, the important questions to be answered by the ARDL model are whether shocks to the system converge back to the estimated long run time path (is the estimated long run model stable?) and whether the inclusion of a quadratic effect for ENPSeat is significant and its estimated shape consistent with the hypothesized role of political competition in relation to government size. For convergence, the error correction term estimate needs to be both negative and significantly less than one. Table 1 indicates this for all cases.[18] The estimated error correction terms, however, are relatively small in absolute size indicating that the time frame for correction back to the long run can be as long as five years. The preferred pooled mean regression model estimate suggests a somewhat longer period of adjustment than the other two.

The coefficients bolded in Table 1 indicate the effect of ENPSeat on government size. Both sets of coefficients in the preferred case of column (2) are highly significant, and the negative/positive sequencing of values indicates a U-shaped relationship with GovSize. This is

[18] The alternative ARDL models are presented for this case only. In the cases that follow only the alternative indicated by the Hausman test and its time trend included version are shown. A full set of results including the Hausman alternative tests are available on request.

consistent with an increase in effective party numbers first enhancing political competition and lowering GovSize before further increases result in greater party fragmentation, less party competition and a decrease in the ability of parties to effectively monitor agency costs. The model's estimate of the efficient level of party competitiveness is when ENPSeats = 4.5. The results in column (4) indicate that the inclusion of a time trend into the preferred *pmg* model increases the Log Likelihood statistic indicating some improvement in the goodness of fit. Additionally, accounting for the time trend in the data results in a marginal increase in the size of the error correction term, suggesting a somewhat faster rate of convergence to the long run, and also reduces the estimate of the efficient level of party competitiveness to ENPSeats = 3.8. Among the control variables, interestingly increasing real per capita income which is considered as a proxy for development has a negative and significant effect on expenditure size across all models suggesting evidence for convergence to a long run stationary level of public spending. Aging population leads to smaller public sector size, whereas growing urbanization is associated with a larger public sector. Increasing literacy and constituency density appear to expand government spending, however the inclusion of time trend in column (4) suggests the earlier result is part of a common temporal trend.

The fractional polynomial model (as specified in equation (23)) is next used to investigate whether the U-shape indicated by the results in columns (2) and (4) have the symmetric shape implied by the quadratic fit. The results of the test and the estimates of the best fitting model are presented in Table 2 together with a graph of its estimated form (and 95% confidence interval) in Figure 1.[19] The results show (a) that monotonicity is rejected relative to non-monotonicity and (b) that the best fitting relationship between government size and ENPSeats has an inverted U-shape that is significantly skewed to the right and reaches a minimum somewhat about the quadratic minimum of 4.5. The analysis is then broadly consistent with the Duverger-Demsetz view of political party competition but with an optimal degree of interparty competition on government size arising at a value much larger than 2.

-- insert Table 2 and Figure 1 about here –

[19] To economize on space we present only the *fp* graph in future cases. The full *fp* analysis is available on request.

In Table 3 we use ARDL analysis to test the model's prediction that the aggregate tax rate (aggregate state revenues as a share of state GDP) will have a similar U-shaped relationship with ENP(Seats). Agency costs are expected to first fall and then rise with an increase in ENP requiring a similar response in the unit tax rate. In this case the Hausman specification test again indicates the pooled mean group model is the preferred version. Two representations of the *pmg* model are shown in Table 3. The model in column (2) includes a correction for the possibility of a time trend unaccounted for by the model's other variables.

-- insert Table 3 and Figure 2 about here --

As was the case for Govsize, the model's coefficient estimates for Tax rate are consistent with long run convergence (at a similar slow rate) and with the hypothesized U-shaped relationship with ENP(Seats). Once again the analysis implies an efficient level of party competition at an ENPSeat value (4.5) above the value that would be expected under Duverger-Demsetz. The inclusion of the time trend in column (2) reduces marginally the ENPSeat value at which the Tax rate is minimized and improves somewhat the fit of the model with the control variables. In terms of institutional effects, the data suggests that a shift to the more efficient VAT tax system has improve revenue collection but that suggestion becomes significant only when temporal dimension of the data in column (2) is taken into account. Finally we note one seemingly paradoxical result. That is, from Table 2 it can be seen that larger grants increase the average tax rate $\left(\frac{T}{Y}\right)$. This seeming anomaly can be explained, however, by the fact that central government grants increase state government revenues (i.e., T) without the corresponding need to increase the state tax rate (t).

Figure 2 presents the graph of the best fitting fractional polynomial and like the case of GovSize indicates a U-shaped relationship for the Tax rate case that is asymmetric about its minimum and skewed rather strongly to the right. This implies that tax rate response to the rapid rise in effective party competitiveness is faster than the fall as ENPSeats passes the minimum point and political party structure becomes increasingly fragmented.

Stage 2 test: The composition effect: (GCons_ratio) and ENP(Seat)

The model in section 2 also predicts that the composition of government output is affected by political competition such that the share of consumption services will rise with more effective party competition. This implies that ratio of government consumption to investment services (GCons_ratio) will rise with an initial increase in ENPSeats before further increases in ENPSeats result in party fragmentation reverse the electoral incentive facing the governing party. ARDL modelling is again used to test this prediction and in this case the Hausman specification test indicates that the pooled mean group alternative (holding constant the long run coefficients) is the preferred version. Two versions of the ARDL *pmg* results for the effect of ENP(Seat) on GCons_ratio are presented in Table 4: the first with fixed (state) effects and the second adding a correction for the possibility of a time trend. The results are consistent with a stable long run relationship (the error correction terms are both significantly less than one) and the relatively large size of the coefficient estimates indicates less than a two year transitional adjustment to the long run. Note that the presence of a fiscal rule found elsewhere to add fiscal accountability to state budgets (Chakraborty and Dash, 2017) is here found consistent with decreasing (increasing) the consumption (investment) share of state budgets. Increases in intergovernmental transfers are found to shift the composition of state spending towards public consumption while increasing urbanization shifts the composition towards public investment.

-- insert Table 4 and Figure 3 about here --

For our purposes the quadratic results suggest an inverted U-shaped relationship between GCons_ratio and ENP(Seats) that peaks at an ENPSeat value of about 3. In this form the hypothesized relationship receives only weak support from the data. However with the more general use of fractional polynomial analysis, the fit is improved. Figure 3 presents the graph of the best fitting fractional polynomial and indicates an inverted U-shaped relationship that peaks closer to ENPSeats = 4 and is skewed somewhat to the left.[20] This implies that compositional

[20] Best fitting fractional polynomial regression: F(10,13) = 51.6; Prob > F = 0.000
LnGCons_ratio = 1.99** - .005Rypc - .409DOld - .017Literacy - .0.001Density - .522 Fiscal Rule*** - .018Urban - .039***DGovsize + .047**Time trend + .048**ENP_Seat_1 - .008***ENP_Seat_2.

response to rising party competition occurs quite slowly and falls off faster after it passes its peak. While this is consistent with the model's predicted effect, it also implies that the electoral advantage of offering ever larger consumption alternatives wears off quickly as a winning party strategy. As party fragmentation increases, the benefit of responding to widening variety of policies targeting ever small segments of the electorate falls quickly relative to alternatives favoring growth.

Stage 3 Test: The effect of ENP(Seat) on Growth

The third step is to test the prediction that ENPSeats, as a reflection of political party competition, has an inverted U-shaped relationship with the growth rate of real per capita income (Growth). In this case because both covariates are stationary, I(0), we adjust our control variables for stationarity and run two versions of our test: the first, a dynamic fixed effects panel model (as specified in equation (24)) and the second, a two stage least squares (2SLS) version using as instruments the lagged values of ENPSeats and fiscal variables to help account for endogeneity.[21] The results are presented in Table 5. Both equations include a lagged dependent to account for persistence in the growth rate where their coefficient estimates indicate slow convergence (-.072 and -.074) back to the period average growth rate of 3.5%. The two models explain roughly fifty percent of the variation in state growth rates and highlight the roles played by urbanization and agriculture in contributing to growth. The insignificance of Reservation, a proxy for the political advantage given to socially and geographically disadvantaged groups, suggest that this form of affirmative action has not played a significant role in increasing the growth rate of Indian states. Surprisingly changes in the level of literacy have also been found to be an insignificant factor in increasing economic growth.

-- insert Table 5 about here --

In terms of our variables of interest, ENP(Seats) does exhibit the predicted inverted U-shaped relationship with real per capita growth. The sequencing of significant positive and negative coefficients is consistent with increases in effective party numbers increasing party

---

[21] Fiscal outcomes are used as instruments, rather than explanatory variables, because our theoretical prediction has ENP affect economic growth through all the channels that different fiscal policies create under changing competitive conditions represented by the changes in political party structure.

competitiveness and through this the growth rate before peaking in its effect at an ENP(Seat) value of about 3.5. Beyond that level further increases in ENP(Seat) are associated with increased party fragmentation, consistent with the hypothesis that a reduction in the intensity of political party competition undermines the growth rate.

Finally we again use fractional polynomial (fp) analysis to plot the best fitting relationship under more general forms of non-monotonicity. The graph of the best fitting second degree fractional polynomial of the growth rate then is presented in Figure 4. It can be seen to be only mildly asymmetric with the growth rate peaking and an ENPSeat value in the range of the 3.5 found for the quadratic case.

--insert Figure 4 about here --

From a policy perspective, because the average value of ENPSeats across our 14 Indian states is 2.75 with a standard deviation of 1.05, our analysis implies that there are at least some states that would have benefited from a larger ENPSeats and greater effective party competition while others with a high ENPSeat value may have suffered from excessive fragmentation. In the latter cases state rates of economic growth would benefit from a fall in party numbers. In short, either of these extremes implies a level of effective party competition biasing policy towards shorter term electoral advantage and away from longer term growth objectives.

5. Conclusion

In this paper we have argued that changes in the effective number of political parties (ENP) reflect underlying changes in the form and intensity of political party competition that have implications for government's size, output composition and rate of economic growth. The key hypothesis is that ENP is a non-monotonic measure of party competition such that increases in ENP from a low level breakdown oligopolistic party behavior arising at the expense of the electorate. At some point, however, further increases in ENP peak in their competitive effectiveness by fragmenting rival party electoral credibility, reducing effective competition and allowing the governing party to benefit at the expense of voters. In the model that begins the paper increases in ENP that increase party competitiveness also requires an electoral strategy that substitutes consumption for government investment services. The increase in agency costs and associated decline in the quality of governance leads to a concomitant rise in taxes, decline in complementary private investment, and hence lower growth.

The empirical section tests for the channels by which ENP is hypothesized to affect economic growth through political competition. First, we find the data are consistent with the hypothesis that, controlling for demographic and other influences on government size, increases in ENP do have a non-monotonic U-shaped effect on government size and the tax share of state income. Although the relationship is somewhat weaker in the second test, an increase in ENP is found to have an inverted U-shaped effect on the consumption to investment ratio of government services.[22] Third, we test for the effect of ENPSeat on growth and find that the data are consistent with the predicted inverted U-shape, consistent with the hypothesis that political party competitiveness is an effective enhancer of economic growth.

To the extent that ENP is a non-monotonic measure of the effects of political party competition on government's size and growth, our analysis suggests that political party competitiveness peaks somewhere in the range of 3 to 4.5. Depending upon the specifics associated with each state, party competitive is maximized at an ENPSeat value larger than that implied by Duverger-Demsetz analysis. Finally our analysis does offer a word of caution for institutional innovations that offer wider representation at the cost of party structure fragmentation. Greater representation in the legislature need not mean greater representation in policy and a wider span of government consumption services across heterogeneous communities may come at the cost of foregoing the common gains through higher future incomes.

[22] One feature that may help account for the relative weakness of the consumption mix hypothesis is that for Indian states the greater visibility of development infrastructure projects means that larger pre-election investment spending appears to work well as an election strategy. See Ferris and Dash (2019).

Data Sources

The panel data used cover 14 major Indian states: Andhra Pradesh, Bihar, Gujarat, Haryana, Karnataka, Kerala, Madhya Pradesh, Maharashtra, Odisha, Punjab, Rajasthan, Tamil Nadu, Uttar Pradesh, and West Bengal and cover the fiscal years from 1959-60 to 2019-20.

Public finance variables: The *Reserve Bank of India Bulletin* provides the longest time-series public finance data at the state level. All expenditure variables are net of interest. Various issues of the *RBI Bulletin* were used to collate this dataset.

Political variables: The *Election Commission of India* (*ECI*) publishes details of both parliamentary and assembly elections on their website (http://eci.nic.in/eci/eci.html). Information available in *ECI's* reports is used to prepare the coding of the qualitative variables: election year, political alignment, party names, seat shares.

Economic and demographic variables: Data for these variables are obtained from the *National Accounts Statistics*. Time-series data for variable state domestic product in constant prices (2004-05 rupees) is not readily available for the entire period. The base year changes approximately once in every decade, and the method of back-ward splicing is used to account for base year adjustment.

Descriptive Statistics for 14 Indian States: 1959/60 – 2019/20

Variable name	Definition	Obs.	Mean	Standard Deviation	Fisher Test for panel unit root
GovSize	Noninterest aggregate state expenditure/state GDP	831	15.64	5.06	χ^2= 30.17 Prob =.355 D(.) χ^2=639 Prob = 0
Tax_rate	total revenue receipts as percentage of state domestic product	831	13.01	4.34	χ^2= 46.77 Prob =.015 D(.) χ^2=623 Prob = 0
GCons_ratio	Govt revenue (consumption)/capital expenditures	831	.021	.012	χ^2= 90.68 Prob = 0
ENPSeat	1 divided by the sum of party seat shares squared	841	2.75	1.05	χ^2= 124.3 Prob = 0
Rypc	Real state income per capita (1000's)	830	21.11	17.39	χ^2 = 0 Prob = 1 D(.) χ^2=57.1 Prob= .0009
Grypc	Growth rate of real state income per capita	816	.035	.069	χ^2=365.3 Prob = 0
Density	Population size per square kilometer (in 1000's)	831	365.84	246.13	χ^2= 44.9 Prob = .02 D(.)χ^2= 159.9 Prob = 0
Old	Percentage of the state population over 60	806	7.12	1.43	χ^2= 2.00 Prob = 1 D2(.) χ^2= 158.1 Prob = 0
Literacy	Percentage of the state population that is literate	806	54.03	19.43	χ^2= 20.1 Prob .86 D(.)χ^2= 47.9 Prob = .01
Urbanization	Percentage of the state population living in urban areas.	806	26.07	9.75	χ^2=2.19 Prob = 1 D2(.) χ^2= 40.1 Prob = .06
Fiscal Rule	1 if a fiscal rule adopted, 0 otherwise	846	.255	.436	
VATdummy	1 if Value added tax is in place, 0 otherwise. Adopted between 2003-2008	846	.241	.428	
Reservation	(reserved seats/assembly size)*100	844	22.4	7.6	χ^2=122.8 Prob = 0
Agriculture share	Agriculture's share of state GDP	831	36.36	14.12	χ^2 = 17.6 Prob .93 D(.)χ^2= 463.3 Prob = 0
Grant size	Intergovernmental transfers/ noninterest government expenditure	846	13.71	5.96	χ^2= 116.3 Prob = 0

D(.) {D2(.)} first and second difference operators.

Table 1
Alternative Autoregressive Distributed Lag Models of Ln(GovSize)
14 Indian States: 1959/60 – 2019/20
(standard errors in brackets)

Error Correction Form	Dynamic Fixed Effects Regression (state clustered) (1)	**Pooled Mean Group Regression (2)**	Mean Group Regression (3)	Pooled Mean Group Regression (with Time trend) (4)
Long Run:				
Real income per capita (Rypc in 1000s)	-.015*** (.003)	-.015*** (.002)	-.020*** (.007)	-.017*** (.002)
D(Old)	-.699 (1.00)	-1.45*** (.386)	-1.16 (.773)	-.806*** (.312)
Literacy	.011*** (.003)	.006** (.003)	.016 (.002)	-.014*** (.005)
Density	.0005** (.0002)	.001*** (.0001)	-.002 (.002)	-.0004** (.0002)
Fiscal Rule	.048 (.056)	.029 (.043)	.044 (.054)	.062 (.043)
Urbanization	.043*** (.009)	.067*** (.009)	.115*** (.035)	.016*** (.007)
Grant size	.004 (.006)	.002 (.003)	.004 (.003)	.003 (.003)
ENPSeat	**-.320*** (.088)**	**-.214*** (.057)**	**-.105 (.165)**	**-.214*** (.049)**
ENPSeat_squared	**.040*** (.010)**	**.024*** (.009)**	**.010 (.033)**	**.028*** (.007)**
Time trend				.040*** (.006)
Short Run:				
Error Correction Term	-.213*** (.021)	-.301*** (.046)	-.665*** (.051)	-.34*** (.035)
Growth rate of real income per capita (grypc)	-.714*** (.069)	-.684*** (.138)	-.491*** (.103)	-.676*** (.145)
D2(Old)	-.003 (.182)	.158 (.254)	.191 (.382)	-.277 (.246)
D(Density)	-.0006*** (.0001)	-.0005 (.004)	-.004 (.006)	-.003 (.005)
D(Urban)	-.007 (.020)	.070 (.053)	.002 (.104)	.046 (.052)
D(Grant_size)	-.002 (.002)	-.003 (.002)	-.003 (.002)	-.004** (.002)
Constant	.351 (.056)	.307*** (.096)	.175 (.433)	.853*** (.090)
Observations	778	778	778	778
Fixed Effects	Yes	Yes	Yes	Yes
Log Likelihood		791.8		806.9
ENP(Seat) value that minimizes government size	**4.00**	**4.46**	**5.25**	**3.82**

* (**)[***] indicates significantly different from zero at 10% (5%) [1%]. D(.){D2(.)} first {second} difference operator.

Table 2

Fractional Polynomial Regression (comparison of 44 fitted models)

ENP(seat)	Test Df	Deviance	Residual std. dev.	Deviance difference	P	Powers
Omitted	0	-514.2	0.176	56.43	.000	
Linear	1	-533.4	0.174	37.17	.000	1
m = 1	2	-548.3	0.173	22.25	.000	-1
m = 2	4	-570.6	0.171	0.00		2 3

Test df is degrees of freedom, and P = P > F is significance level for tests comparing models vs. model with m = 2 based on deviance difference, F(df, 762). Below *(**)[***] report significance at 10%(5%)[1%]

Best fitting regression: F(9,13) = 86.4; Prob > F = 0.000; xtFisher on equation residual = 73.3 Prob = 0

Ln(GovSize) = 1.34*** - .012***Rypc - .012 Old - .011Literacy*** + .0.0004**Density - .091**Fiscal Rule

+ .037**Urban + .0002Grant_size - .034***ENP_Seat_1 + .005***ENP_Seat_2

Figure 1

Component plot of best fitting fractional polynomial of noninterest Government Expenditure Size (with 95% confidence interval)

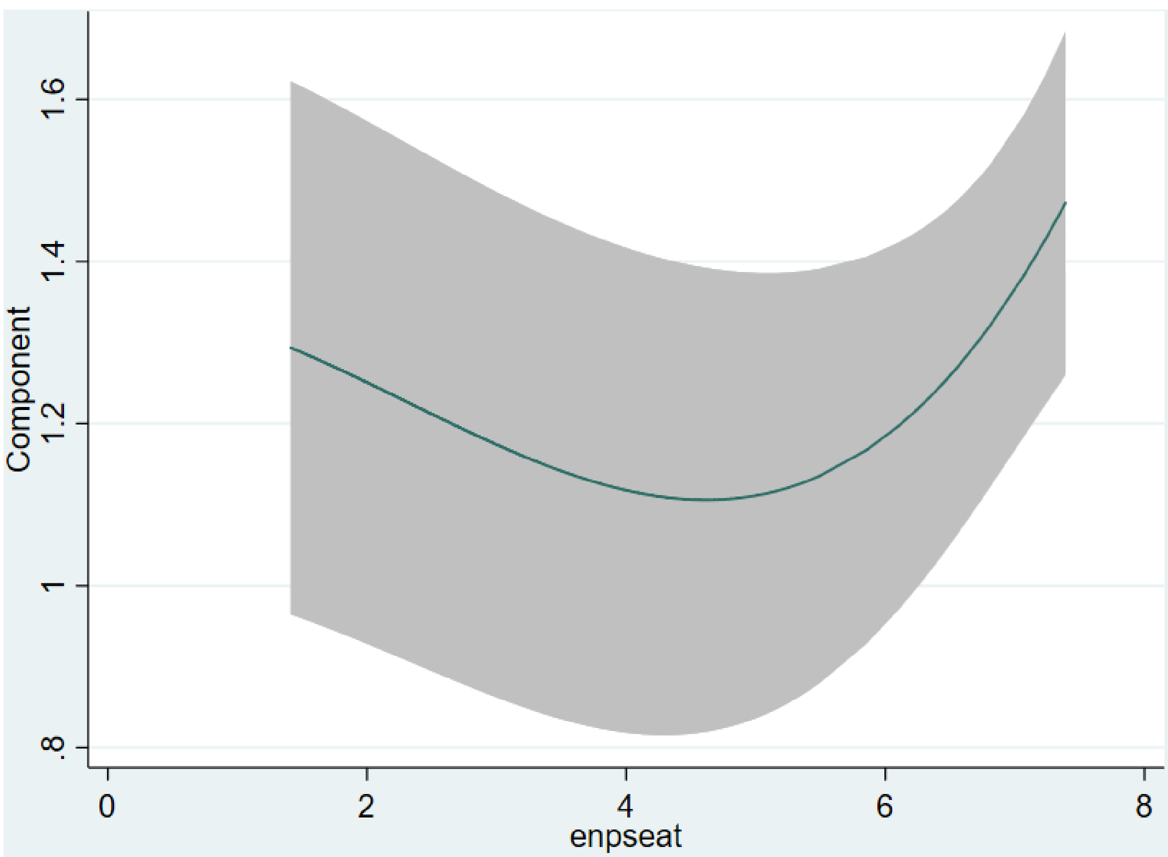


Table 3

Autoregressive Distributed Lag Model of the Ln(Tax rate)
14 Indian States: 1959/60 – 2019/20
(robust standard errors in brackets)

Error Correction Form	Pooled Mean Group Regression (1)	Pooled Mean Group Regression (with Time trend) (2)
Long Run:		
Real income per capita	-.016*** (.002)	-.019*** (.002)
D(Old)	.044 (3.47)	-.404 (.277)
Literacy	.016*** (.002)	-.014*** (.005)
Density	.0002 (.0002)	-.0005*** (.0001)
VAT Dummy	.038 (.048)	.101** (.046)
Urbanization	.032*** (.007)	.014* (.007)
Grant size	.011*** (.003)	.012*** (.003)
ENPSeat	**-.237*** (.039)**	**-.162*** (.050)**
ENPSeat_squared	**.026*** (.006)**	**.019*** (.007)**
Time trend		.041*** (.005)
Short Run:		
Error Correction Term	**-.260*** (.036)**	**-.306*** (.035)**
Growth rate of real income per capita	-.685*** (.122)	-.669*** (.133)
D2(Old)	-.280 (.184)	-.238 (.196)
D(Density)	.001 (.003)	.0006 (.004)
D(Urban)	.045 (.038)	.023 (.033)
D(Grant_size)	.004*** (.001)	.004*** (.013)
Constant	.382*** (.074)	.729*** (.089)
Observations	778	778
Fixed Effects	Yes	Yes
Log Likelihood	893.1	902.3
ENP(Seat) when tax rate minimizes	**4.5**	**4.3**

* (**)[***] indicates significantly different from zero at 10% (5%) [1%]. D(.) [D2()] first [second] difference operator.

Figure 2
Component plot of best fitting fractional polynomial of Ln(Tax rate)
(with 95% confidence interval)

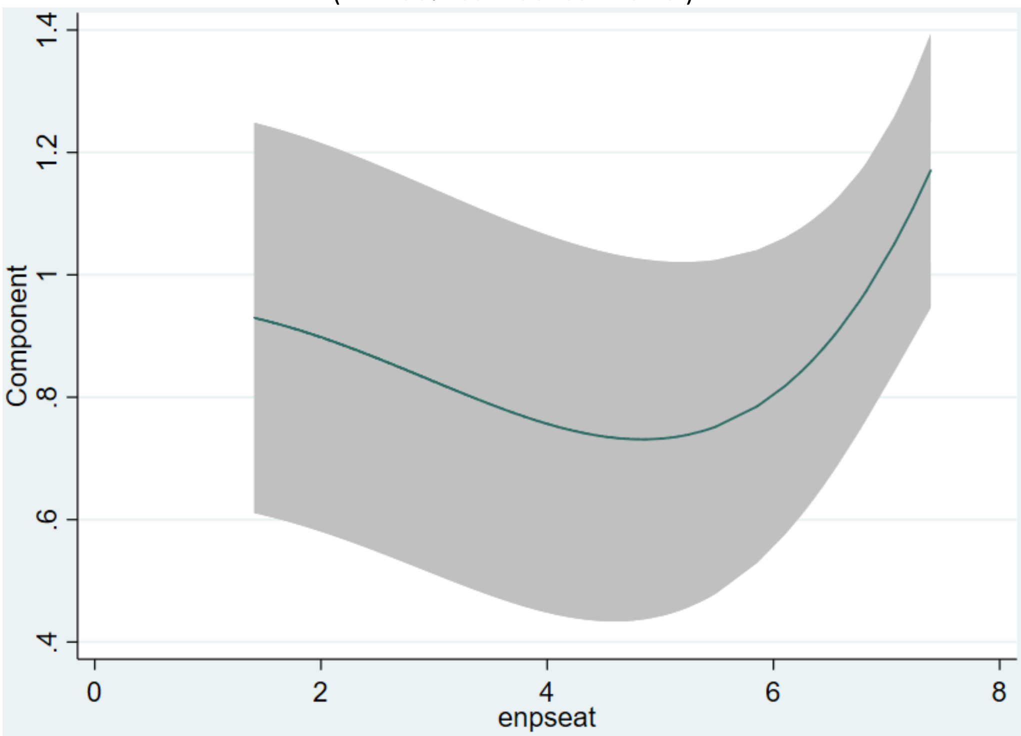


Figure 3
Component plot of best fitting fractional polynomial of Ln(GCons_Ratio)
(with 95% confidence interval)

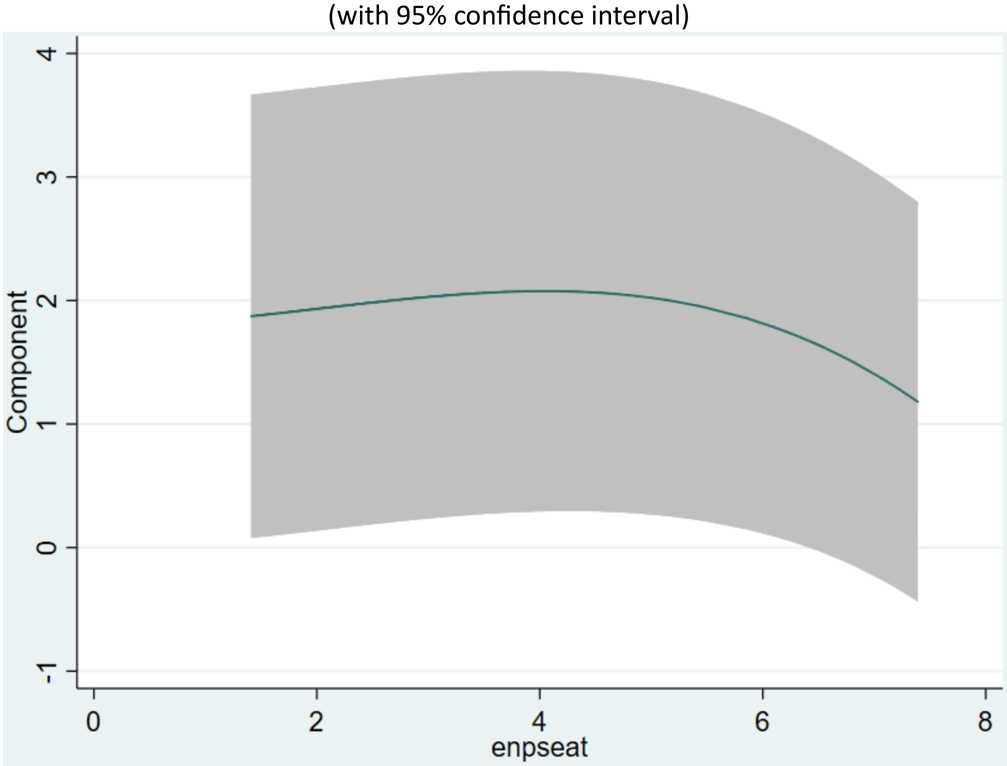


Table 4
ARDL Model of the Ln(GCons_Ratio)
14 Indian States: 1959/60 – 2019/20
(robust standard errors in brackets)

Error Correction Form	Pooled Mean Group Regression (2)	Pooled Mean Group Regression (with Time trend) (3)
Long Run: Real income per capita	.006* (.003)	.001 (.003)
D(Old)	-.734 (.595)	-.248 (.671)
Literacy	.027*** (.005)	-.011 (.009)
Density	-.0004 (.0003)	-.001*** (.0003)
Fiscal Rule Dummy	-.596*** (.092)	-.499*** (.089)
Urbanization	-.030** (.014)	-.057*** (.016)
Grant size	.020*** (.007)	.013** (.006)
ENPSeat	**.179*** **(.104)**	**.143** **(.099)**
ENPSeat_squared	**-.029**** **(.0014)**	**-.025*** **(.014)**
Time trend		.050*** (.012)
Short Run: Error Correction Term	-.604*** (.061)	-.621*** (.063)
Growth rate of real income per capita	-.154 (.277)	-.238 (.292)
D2(Old)	1.22 (.867)	1.33 (.917)
D(Density)	.021 (.023)	.022 (.025)
D(Urban)	-.122 (.144)	-.204 (.146)
D(Grant_size)	-.001 (.004)	.001 (.004)
Constant	.485*** (.105)	1.57*** (.226)
Observations Fixed Effects Log Likelihood ENP(Seat) when Gons_Ratio bottoms	778 Yes -184.3 3.09	778 Yes -177.3 2.9

* (**)[***] indicates significantly different from zero at 10% (5%) [1%]. D(.) [D2()] first [second] difference operator.

Table 5

Panel Data effects of ENP(Seat) and the Growth Rate of State per capital Output
14 Indian States: 1959/60 – 2019/20
(standard error adjusted for clusters in brackets)

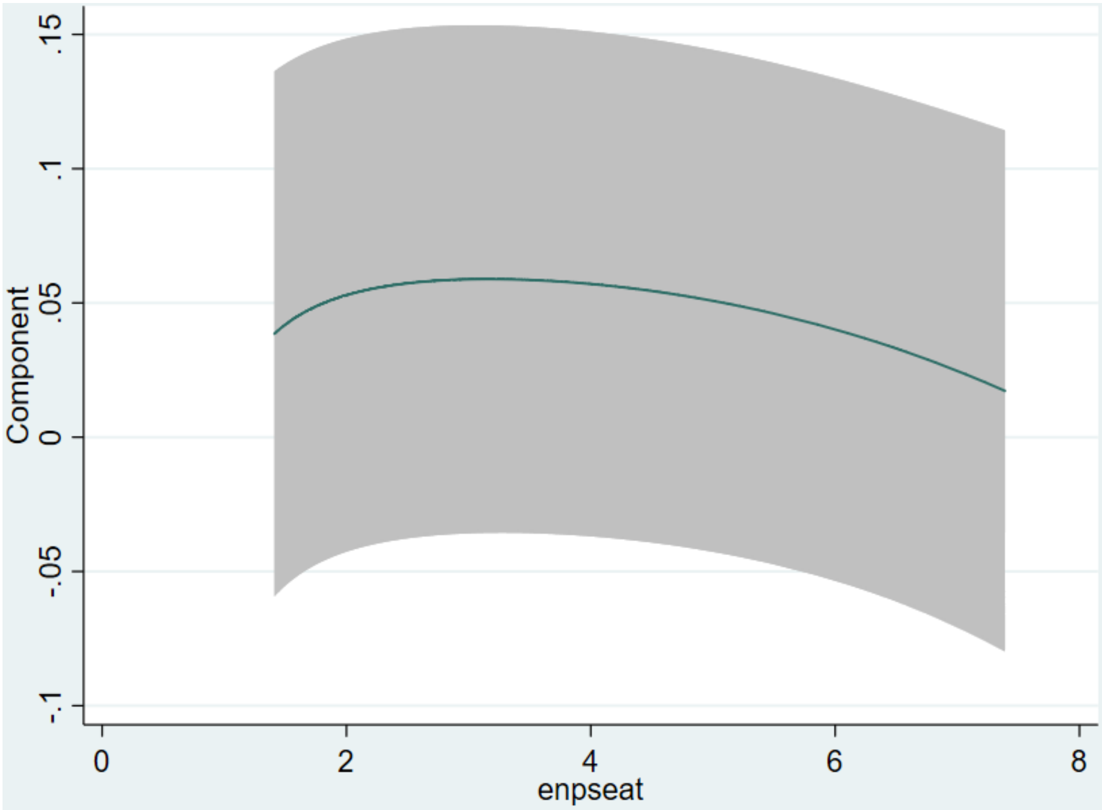
	Fixed Effects Growth Rate of Income per capita (1)	Two Stage Least Squares Growth Rate of Income per capita (2)
Lagged_Growth Rate (grypc)	-.072*** (.023)	-.074*** (.028)
D(Literacy)	.003 (.003)	.003 (.003)
D(Urban)	.026*** (.006)	.025*** (.008)
D(Agriculture_share)	.014*** (.001)	.014*** (.001)
Reservation	-.001 (.002)	-.001 (.001)
ENPSeat	**.022*** (.008)**	**.027*** (.009)**
ENPSeat_squared	**-.003*** (.0007)**	**-.004*** (.001)**
Constant	.020 (.052)	.012 (.028)
Statistics		
Number of Obs.	777	776
Fixed Effects	Yes	Yes
Overall R^2	.485	.485
F	52.0***	1087.8***
ENP(Seats) peak	3.58	3.51

*(**)[***], significantly different from zero at 10% (5%) [1%]. D(.) first difference.

Instruments in 2SLS model: L.grypc dliteracy durban dagriculture_share reservation L.ENP(Seat) L.ENPSeat_sq L.dtaxrate L.dgovsize

Figure 4

Component plot of best fitting fractional polynomial of State Per Capita Growth Rate (with 95% confidence interval)



References

Acemoglu, D., & J.A. Robinson (2006). De Facto Political Power and Institutional Persistence. *American Economic Review*, 96(2): 325-330.

Baraldi, A. L. (2008). Effects of Electoral Rules, Political Competition and Corruption on the Size and Composition of Government Consumption Spending: An Italian Regional Analysis. *The B.E. Journal of Economic Analysis & Policy* 8(1).

Barro, R. J., & X. Sala-i-Martin (2004). *Economic growth*. MIT Press.

Blackburne III, E.F., & M.W. Frank (2007). Estimation of nonstationary heterogeneous panels. *Stata Journal* 7(2): 197-208.

Carmeci, G., L. Mauro & F. Privileggi (2019). Growth maximizing government size, social capital, and corruption. *Journal of Public Economic Theory* 23: 438-461.

Chakraborty, P., and B.B. Dash (2017). Fiscal reforms, fiscal rule, and development spending: How Indian states have performed? *Public Budgeting & Finance* 37(4): 111-133.

Chhibber, P.K., & I. Nooruddin (2004). Do party systems count? The number of parties and government performance in the Indian states. *Comparative Political Studies* 37: 152-187.

Chhibber, P.K., & K. Kollman (1998). Party Aggregation and the Number of Parties in India and the United States. *American Political Science Review* 92(2): 329-342.

Chhibber, P.K., & K. Kollman (2004). *The formation of national party systems.* Princeton: Princeton University Press.

Dash, B.B., & A.V. Raja (2013) Do political determinants affect the size and composition of public expenditure? A study of the Indian states. *International Review of Economics* 60(3): 293-317.

Dash, B.B., J.S. Ferris & S.L. Winer (2019) The measurement of electoral competition, with application to Indian states. *Electoral Studies* 62, 102070.

Demsetz, H. (1968). Why regulate utilities? *Journal of Law and Economics*, 11(1): 55-65.

Demsetz, H. (2008). The Contrast Between Firms and Political Parties. In H. Demsetz, *From economic man to economic system: Essays on human behavior and the institutions of capitalism.* Cambridge University Press: 130-140.

Durham, J.B. (1999). Economic Growth and Political Regimes. *Journal of Economic Growth* 4, 81–111.

Duverger, M. (1954). *Political Parties: Their Organization and Activity in the Modern State*. Methuen and Co.

Ferris, J.S., & B.B. Dash (2019). Expenditure visibility and voter memory: a compositional approach to the political budget cycle in Indian states, 1959–2012. *Economic Governance* 20: 129–157.

Ferris, J.S. & B.B. Dash (2023). On the structure of the political party system in Indian states, 1957 – 2018. *Constitutional Political Economy* 34(1): 1-25.

Ferris, J.S., & B.B. Dash (2024). Between election rivalry and the agency costs of government: The effectiveness of party competition across Indian States, 1957 – 2018. *Public Finance Review* 52(3): 314-344.

Ferris, J.S., & M.C. Voia (2023). Do rival political parties enforce government efficiency? Evidence from Canada, 1867 – 2021. *European Journal of Political Economy* 77(2), 102313.

Ferris, J.S., S.L. Winer & B. Grofman (2016). The Duverger-Demsetz Perspective on Electoral Competitiveness and Fragmentation: with application to the Canadian Parliamentary System, 1867 – 2011, in Gallego, M. and N. Schonfield (eds.), *The Political Economy of Social Choices*, 2016, New York: Springer Publishing, pp. 93-122.

Gwartney, J. D., R.G. Holcombe & R.A. Lawson (2006). Institutions and the impact of investment on growth. *Kyklos* 59(2): 255-273.

Leonida, L., D.M.A. Patti, & P. Navarra, (2013). Testing the political replacement effect: A panel data analysis. *Oxford Bulletin of Economics and Statistics* 75(6): 785-805.

Leonida, L., D.M.A. Patti, A. Marini & P. Navarra (2015). Political competition and economic growth: A test of two tales. *Economic Letters* 135: 96-99.

Lewis, B.D., & A. Hendrawan (2019). The impact of majority coalitions on local government spending, service delivery and corruption in Indonesia. *European Journal of Political Economy* 58(2): 178-191.

Mendilow, J. (1992). Public party funding and party transformation in multiparty systems. *Comparative Political Studies* 25(1): 90-117.

Pesaran, M.H., & R.P. Smith (1995). Estimating long-run relationships from dynamic heterogeneous panels. *Journal of Econometrics* 68: 79-113.

Pesaran, M.H., Y. Shin & R.P. Smith (1999). Pooled mean group estimation of dynamic heterogeneous panels. *Journal of the American Statistical Association* 94: 621-634.

Scartascini, C.G., & W.M. Crain (2021). The Size and Composition of Government Spending in Multi-Party Systems. In: Hall, J. & B. Khoo (eds) *Essays on Government Growth. Studies in Public Choice,* vol 40. Springer, Cham.

Winer, S.L., Ferris, J.S., Dash, B.B. & Chakraborty, P. (2021) Political competitiveness and the private-public structure of public expenditure: A model and empirics for the Indian States" *International Tax and Public Finance* 28(6): 1430-1471.

Solving the model's comparative statics

Beginning from the set of first order conditions:

$$\frac{\gamma}{g_t(N_t)} - \mu = 0 \quad (1)$$

$$(1-\tau)\omega\theta k_t^{\alpha} K_{g_t}^{\omega-1} - \frac{1}{L} = 0 \qquad \text{or} \qquad \frac{\omega y_t}{K_{g_t}} = \frac{1}{L} \quad (2)$$

$$-\frac{2}{(1-\tau)} + \mu y_t = 0 \qquad \text{or} \qquad \frac{2}{(1-\tau)} = \mu y_t \quad (3)$$

$$\tau y_t - g_t(N_t) - \frac{K_{g_t}}{L} - z(N_t) = 0 \quad (4)$$

Total differentiation of the FOC's with respect to N yields (where $y_t = (1-\tau)\theta k_t^{\alpha} K_{g_t}^{\omega}$)

$$-\frac{\gamma}{g^2} dg - \frac{\gamma}{g^2} g_N dN - d\mu = 0$$

$$-\omega\theta k_t^{\alpha} K_{g_t}^{\omega-1} d\tau + (1-\tau)\omega(\omega-1)\theta k_t^{\alpha} K_{g_t}^{\omega-2} dK_g = 0$$

$$-\frac{2}{(1-\tau)^2} d\tau + y_t d\mu - \mu\theta k_t^{\alpha} K_{g_t}^{\omega} d\tau + \mu(1-\tau)\omega\theta k_t^{\alpha} K_{g_t}^{\omega-1} dK_g = 0$$

$$y_t d\tau - \tau\theta k_t^{\alpha} K_{g_t}^{\omega} d\tau + \tau(1-\tau)\omega\theta k_t^{\alpha} K_{g_t}^{\omega-1} dK_g - 1dg - g_N d_N - \frac{1}{L} dK_g - z_N dN = 0$$

Rewriting this in matrix form (and then using the FOCs to simplify the terms):

$$\begin{bmatrix} -\frac{\gamma}{g^2} & 0 & 0 & -1 \\ 0 & (1-\tau)\omega(\omega-1)\theta k_t^{\alpha} K_{g_t}^{\omega-2} & -\omega\theta k_t^{\alpha} K_{g_t}^{\omega-1} & 0 \\ 0 & \mu(1-\tau)\omega\theta k_t^{\alpha} K_{g_t}^{\omega-1} & -\frac{2}{(1-\tau)^2} - \mu\theta k_t^{\alpha} K_{g_t}^{\omega} & y_t \\ -1 & \tau(1-\tau)\omega\theta k_t^{\alpha} K_{g_t}^{\omega-1} - \frac{1}{L} & y_t - \tau\theta k_t^{\alpha} K_{g_t}^{\omega} & 0 \end{bmatrix} \begin{bmatrix} dg/dN \\ dK_g/dN \\ d\tau/dN \\ d\mu/dN \end{bmatrix} = \begin{bmatrix} \frac{\gamma g_N}{g^2} \\ 0 \\ 0 \\ g_N + z_N \end{bmatrix}$$

$$\begin{bmatrix} -\frac{\gamma}{g^2} & 0 & 0 & -1 \\ 0 & \frac{\omega(\omega-1) y_t K_{g_t}^{-1}}{K_{g_t}} & -\omega\theta k_t^{\alpha} K_{g_t}^{\omega-1} & 0 \\ 0 & \frac{\mu\omega y_t}{K_{g_t}} & -\frac{2}{(1-\tau)^2} - \mu\theta k_t^{\alpha} K_{g_t}^{\omega} & y_t \\ -1 & \frac{\tau\omega y_t}{K_{g_t}} - \frac{1}{L} & \frac{(1-2\tau) y_t}{1-\tau} & 0 \end{bmatrix} \begin{bmatrix} dg/dN \\ dK_g/dN \\ d\tau/dN \\ d\mu/dN \end{bmatrix} = \begin{bmatrix} \frac{\gamma g_N}{g^2} \\ 0 \\ 0 \\ g_N + z_N \end{bmatrix}$$

Using $\frac{\omega y_t}{K_{g_t}} = \frac{1}{L}$ and $\frac{2}{(1-\tau)} = \mu y_t$

$$\begin{bmatrix} -\frac{\gamma}{g^2} & 0 & 0 & -1 \\ 0 & \frac{(\omega-1)K_{g_t}^{-1}}{L} & -\omega\theta k_t^{\alpha} K_{g_t}^{\omega-1} & 0 \\ 0 & \frac{\mu}{L} & -\frac{\mu y_t + \mu y_t}{(1-\tau)} & y_t \\ -1 & \frac{(\tau-1)}{L} & \frac{(1-2\tau)y_t}{1-\tau} & 0 \end{bmatrix} \begin{bmatrix} dg/dN \\ dK_g/dN \\ d\tau/dN \\ d\mu/dN \end{bmatrix} = \begin{bmatrix} \frac{\gamma g_N}{g^2} \\ 0 \\ 0 \\ g_N + z_N \end{bmatrix}$$

$$\begin{bmatrix} -\frac{\gamma}{g^2} & 0 & 0 & -1 \\ 0 & \frac{(\omega-1)K_{g_t}^{-1}}{L} & -\frac{\omega\theta(1-\tau) k_t^{\alpha} K_{g_t}^{\omega-1}}{1-\tau} & 0 \\ 0 & \frac{\mu}{L} & -\frac{2\mu y_t}{(1-\tau)} & y_t \\ -1 & \frac{(\tau-1)}{L} & \frac{(1-2\tau)y_t}{1-\tau} & 0 \end{bmatrix} \begin{bmatrix} dg/dN \\ dK_g/dN \\ d\tau/dN \\ d\mu/dN \end{bmatrix} = \begin{bmatrix} \frac{\gamma g_N}{g^2} \\ 0 \\ 0 \\ g_N + z_N \end{bmatrix}$$

$$\begin{bmatrix} -\frac{\gamma}{g^2} & 0 & 0 & -1 \\ 0 & \frac{(\omega-1)}{LK_{g_t}} & -\frac{1}{(1-\tau)L} & 0 \\ 0 & \frac{\gamma}{g_t L} & -\frac{2\mu y_t}{(1-\tau)} 0 & y_t \\ -1 & \frac{(\tau-1)}{L} & \frac{(1-2\tau)y_t}{1-\tau} & 0 \end{bmatrix} \begin{bmatrix} dg/dN \\ dK_g/dN \\ d\tau/dN \\ d\mu/dN \end{bmatrix} = \begin{bmatrix} \frac{\gamma g_N}{g^2} \\ 0 \\ 0 \\ g_N + z_N \end{bmatrix}$$

$$Det = -\frac{\gamma}{g^2} \begin{vmatrix} \frac{(\omega-1)}{LK_{g_t}} & -\frac{1}{(1-\tau)L} & 0 \\ \frac{\gamma}{g_t L} & -\frac{2\mu y_t}{(1-\tau)} & y_t \\ \frac{(\tau-1)}{L} & \frac{(1-2\tau)y_t}{1-\tau} & 0 \end{vmatrix} + \begin{vmatrix} 0 & \frac{(\omega-1)}{LK_{g_t}} & -\frac{1}{(1-\tau)L} \\ 0 & \frac{\gamma}{g_t L} & -\frac{2\mu y_t}{(1-\tau)} \\ -1 & \frac{(\tau-1)}{L} & \frac{(1-2\tau)y_t}{1-\tau} \end{vmatrix}$$

$$= \frac{\gamma y_t}{g^2} \begin{vmatrix} \frac{(\omega-1)}{LK_{g_t}} & -\frac{1}{(1-\tau)L} \\ \frac{(\tau-1)}{L} & \frac{(1-2\tau)y_t}{1-\tau} \end{vmatrix} - \begin{vmatrix} \frac{(\omega-1)}{LK_{g_t}} & -\frac{1}{(1-\tau)L} \\ \frac{\gamma}{g_t L} & -\frac{2\mu y_t}{(1-\tau)} \end{vmatrix}$$

$$= \frac{\gamma y_t}{g^2} \frac{(\omega-1)}{LK_{g_t}} \frac{(1-2\tau)y_t}{1-\tau} + \frac{\gamma y_t}{g^2} \frac{(\tau-1)}{L} \frac{1}{(1-\tau)L} + \frac{2\mu y_t(\omega-1)}{(1-\tau)LK_{g_t}} - \frac{\gamma}{g_t L} \frac{1}{(1-\tau)L}$$

$$= \frac{\gamma y_t}{g^2} \frac{(\omega-1)}{LK_{g_t}} \frac{(1-2\tau)y_t}{1-\tau} - \frac{\gamma y_t}{g^2 L^2} + \frac{2\mu y_t(\omega-1)}{(1-\tau)LK_{g_t}} - \frac{\gamma}{g_t L} \frac{1}{(1-\tau)L}$$

where all four terms are negative for $\omega < 1$ and $\tau < 1/2$. ($\omega < 1$ by the production technology defined in equation (1)). Assuming $\tau < 1/2$, the determinant is negative satisfying the sufficient condition.

Using Cramer's rule, we solve for the effect of a change in N on our variables:

1. Solving for dK_g/dN, the numerator term is

$$\begin{vmatrix} -\frac{\gamma}{g^2} & \frac{\gamma g_N}{g^2} & 0 & -1 \\ 0 & 0 & -\frac{1}{(1-\tau)L} & 0 \\ 0 & 0 & -\frac{2\mu y_t}{(1-\tau)} & y_t \\ -1 & g_N + z_N & \frac{(1-2\tau)y_t}{1-\tau} & 0 \end{vmatrix}$$

$$= -\frac{\gamma}{g^2}\begin{vmatrix} 0 & -\frac{1}{(1-\tau)L} & 0 \\ 0 & -\frac{2\mu y_t}{(1-\tau)} & y_t \\ g_N + z_N & \frac{(1-2\tau)y_t}{1-\tau} & 0 \end{vmatrix} + \begin{vmatrix} \frac{\gamma g_N}{g^2} & 0 & -1 \\ 0 & -\frac{1}{(1-\tau)L} & 0 \\ 0 & -\frac{2\mu y_t}{(1-\tau)} & y_t \end{vmatrix}$$

$$= -\frac{\gamma}{g^2}(g_N + z_N)\begin{vmatrix} -\frac{1}{(1-\tau)L} & 0 \\ -\frac{2\mu y_t}{(1-\tau)} & y_t \end{vmatrix} + \frac{\gamma g_N}{g^2}\begin{vmatrix} -\frac{1}{(1-\tau)L} & 0 \\ -\frac{2\mu y_t}{(1-\tau)} & y_t \end{vmatrix}$$

$= \frac{\gamma}{g^2}(g_N + z_N)\frac{y_t}{(1-\tau)L} - \frac{\gamma g_N}{g^2}\frac{y_t}{(1-\tau)L} = z_n \frac{\gamma y_t}{g^2(1-\tau)L} > 0$ when $z_N > 0$.

So that when $N > N_{min}$ the numerator is positive and $\frac{dK_g}{dN} = \frac{+ve}{-ve} < 0$. That is an increase in N leads to a decrease in the supply of the government supplied (capital) public good. The opposite is the case if $N < N_{min}$.

2. Solving for $\frac{d\tau}{dN}$, the numerator term is

$$\begin{vmatrix} -\frac{\gamma}{g^2} & 0 & \frac{\gamma g_N}{g^2} & -1 \\ 0 & \frac{(\omega-1)}{LK_{g_t}} & 0 & 0 \\ 0 & \frac{\gamma}{g_t L} & 0 & y_t \\ -1 & \frac{(\tau-1)}{L} & g_N + z_N & 0 \end{vmatrix} = -\frac{\gamma}{g^2}\begin{vmatrix} \frac{(\omega-1)}{LK_{g_t}} & 0 & 0 \\ \frac{\gamma}{g_t L} & 0 & y_t \\ \frac{(\tau-1)}{L} & g_N + z_N & 0 \end{vmatrix} + \begin{vmatrix} 0 & \frac{\gamma g_N}{g^2} & -1 \\ \frac{(\omega-1)}{LK_{g_t}} & 0 & 0 \\ \frac{\gamma}{g_t L} & 0 & y_t \end{vmatrix}$$

$$= \frac{\gamma y_t}{g^2}\begin{vmatrix} \frac{(\omega-1)}{LK_{g_t}} & 0 \\ \frac{(\tau-1)}{L} & g_N + z_N \end{vmatrix} - \frac{\gamma g_N}{g^2}\begin{vmatrix} \frac{(\omega-1)}{LK_{g_t}} & 0 \\ \frac{\gamma}{g_t L} & y_t \end{vmatrix}$$

$$= \frac{\gamma y_t}{g^2}\frac{(\omega-1)}{LK_{g_t}}(g_N + z_N) - \frac{\gamma g_N}{g^2}\frac{(\omega-1)}{LK_{g_t}}y_t = \frac{\gamma(\omega-1)y_t z_N}{g^2 LK_{g_t}} < 0$$

when $z_N > 0$. Hence if $N > N_{min}$, $\frac{d\tau}{dN} = \frac{-ve}{-ve} > 0$ and an increase in the effective number of parties will lead to an increase in the tax rate. In the alternative state where $N < N_{min}$ where an increase in N increases political competition and hence lowers agency costs, then $z_N < 0$ and $\frac{d\tau}{dN} = \frac{+ve}{-ve} < 0$. By breaking the political market power held by few dominant political parties, agency costs allow competition to reduce government costs and allow for a lower tax rate.

3. We can now use these results to solve for the effect of an increase in the effective number of political parties on the growth rate.

From (10) that $\frac{k_{t+1}}{k_t} = \frac{\beta(1-\tau)(1-\alpha)}{(1+\beta)}\left(\frac{y_t}{k_t}\right)$ so that, when $z(N) > 0$ (the fragmentation case)

$$\frac{d\left(\frac{k_{t+1}}{k_t}\right)}{dN} = -\frac{\beta(1-\alpha)}{(1+\beta)}\left(\frac{y_t}{k_t}\right)\left(\frac{d\tau}{dN} > 0\right) + \frac{\beta(1-\tau)(1-\alpha)}{(1+\beta)}\frac{d\left(\frac{y_t}{k_t}\right)}{dN}$$

And since $\frac{y_t}{k_t} = (1-\tau)\theta k_t^{\alpha-1}K_{g_t}^{\omega}$ with k_t given and $z_N > 0$,

$$\frac{d\left(\frac{y_t}{k_t}\right)}{dN} = -\theta k_t^{\alpha-1}K_{g_t}^{\omega}\left(\frac{d\tau}{dN} > 0\right) + \omega(1-\tau)k_t^{\alpha-1}K_{g_t}^{\omega-1}\left(\frac{dKg_t}{dN} < 0\right) < 0.$$

Because both terms are negative, $\frac{d\left(\frac{k_{t+1}}{k_t}\right)}{dN} < 0$. This is the case when $z_N > 0$ and $N > N_{min}$ (excessive political competition). The opposite follows, that is $\frac{d\left(\frac{k_{t+1}}{k_t}\right)}{dN} > 0$, when $z_N < 0$ and $N < N_{min}$ (breaking oligopolistic political power).