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Fostering Growth and Welfare by Determining Market Structure and Inducing Innovation

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Fostering Growth and Welfare by Determining Market Structure and Inducing Innovation*

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Abstract

This paper examines the interplay between market structure, economic growth, and aggregate welfare in a dynamic general equilibrium framework featuring endogenous R&D. The analysis is conducted across three distinct policymaker regimes: free-entry and exit, purely benevolent policymaker, and politically motivated policymaker. In the free-entry regime, market dynamics are governed by the zero-profit condition, resulting in limited firm entry. Under the purely benevolent policymaker regime, market concentration is minimized as the policymaker prioritizes aggregate welfare. The politically motivated policymaker regime introduces a bias toward industry profits, leading to varied market outcomes depending on the weight assigned to producer surplus.

Key findings from the calibration exercise reveal distinct patterns in growth and welfare. While consumer surplus increases with the number of firms, producer surplus

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declines, resulting in an ambiguous welfare trajectory. The purely benevolent policymaker fosters the highest market competition but exhibits the lowest welfare and growth rates due to diminished incentives for R&D. Conversely, the politically motivated policymaker regime achieves the highest growth rates when weight on industry profits is moderate, striking a balance between competition and innovation incentives. However, excessive political bias renders this regime unsustainable, as the equilibrium number of firms falls below the minimum threshold for market viability.

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1 Introduction

Rent-seeking activities are widely acknowledged to reduce economic resources, curtail production, and impede growth. These activities encompass subsidies, tax and tariff evasion, monopoly capture, and corruption. Lobbying represents a prevalent form of rent-seeking, often viewed negatively for channeling resources into unproductive endeavors.

Bhagwati (1982) categorizes a range of rent-seeking activities under the broader concept of "Directly Unproductive Profit-seeking (DUP) Activities." These activities, while yielding pecuniary returns for the individuals or entities involved, do not contribute directly to the production of goods or services that enhance societal welfare. Bhagwati argues that such activities often arise due to distortions in market structures or policy environments, where economic agents allocate resources to secure or maintain advantages—such as monopoly rights, subsidies, or tariffs—rather than to innovate or improve productivity.

One of Bhagwati's key contributions is distinguishing between productive entrepreneurship and rent-seeking behavior. He identifies DUP activities as parasitic to economic growth, as they consume valuable resources (time, effort, and capital) without generating net utility or output. Examples highlighted in his work include lobbying for trade protection, bureaucratic red tape, and corruption in public policy implementation. Bhagwati's framework emphasizes that DUP activities can perpetuate inefficiency by redirecting entrepreneurial energy towards extracting rents rather than fostering competitive markets.

Despite these negative implications, Bhagwati acknowledges that the prevalence of DUP activities often stems from existing institutional or policy constraints, which shape the incentives for such behaviors. His work has become foundational in understanding the broader implications of rent-seeking and has inspired subsequent studies examining how specific forms of DUP activities, such as lobbying, affect innovation, market structure, and welfare.

While Bhagwati (1982) highlights the inefficiencies of rent-seeking activities, subsequent research suggests that their impact is not universally negative. In certain contexts, rent-seeking may play a facilitative role in economic development. Bardhan (1997), for example, provides historical evidence demonstrating how rent-seeking activities have occasionally enabled the rise of entrepreneurial classes. By securing control over resources or navigating institutional barriers, these activities have, at times, acted as catalysts for economic transformation and industrial growth. This perspective highlights the contextual and institutional nuances that determine whether rent-seeking hinders or contributes to economic progress.

Further, the literature indicates that rent-seeking may not always be unproductive, particularly in the context of research and development (R&D)-intensive industries. Grossmann and Steger (2008) highlight the possibility of a positive relationship between R&D investment and incumbent firms' efforts to raise entry barriers for potential competitors. Their analysis demonstrates that anti-competitive behaviors, such as lobbying to increase entry costs or secure exclusive rights, can complement R&D activities. This synergy arises because such behaviors may enhance the profitability of innovation, thereby incentivizing firms to invest more heavily in R&D.

Importantly, the overall effect of these activities on growth and welfare depends critically on the degree of knowledge spillovers. In industries where knowledge spillovers are substantial, these behaviors can amplify innovation's broader benefits, potentially driving higher economic growth and improved welfare. The degree of knowledge spillovers is critical in determining whether these behaviors amplify innovation's broader benefits. High spillovers may allow rent-seeking-driven R&D to have positive externalities, offsetting its anti-competitive effects. Conversely, low spillovers exacerbate market inefficiencies. However, the authors also caution that excessive entry barriers, or lack of spillovers, may negate these positive effects, underscoring the fine balance between fostering innovation and maintaining competitive markets.

Aghion et al. (2005) provide a seminal contribution to the understanding of how market competition influences innovation. Their work builds on the Schumpeterian growth theory, which posits that innovation incentives are closely tied to market structure and firm dynamics. They identify two opposing effects of competition on innovation. First, the escape competition effect suggests that increased competition encourages firms to innovate as a means of outperforming rivals and securing higher profits. This effect is particularly strong for leading firms that already have a competitive edge, as innovation can help them maintain or extend their market dominance.

Conversely, the Schumpeterian effect highlights that excessive competition may reduce innovation incentives for lagging firms. These firms, facing slimmer profit margins and diminished prospects of catching up, may opt out of the innovation race altogether. Aghion et al. (2005) formalize these opposing forces into an inverted-U relationship between competition and innovation, where moderate levels of competition strike the optimal balance, fostering robust innovation incentives across the market.

The authors substantiate their theoretical insights with empirical evidence, using data from the United Kingdom's manufacturing industries. Their findings confirm that industries with moderate competition exhibit the highest rates of innovation, while those with very low or very high competition lag behind. Additionally, they explore the role of institutional factors, such as intellectual property rights and market entry barriers, in mediating the competition-innovation relationship. These factors can either amplify or dampen the effects of competition, depending on their design and enforcement.

Aghion et al.'s work has significant policy implications, suggesting that neither excessive deregulation nor excessive protectionism fosters innovation. Instead, policies should aim to maintain moderate levels of competition, carefully balancing market incentives to encourage innovation without undermining laggard firms' participation in R&D efforts.

Brou and Ruta (2007) extend the analysis of rent-seeking by explicitly incorporating market structure considerations into their models. Their work demonstrates how lobbying for regulatory advantages—such as entry barriers or exclusive rights—can influence both the number of firms in a market and the intensity of innovation within an industry. By increasing entry costs, incumbent firms engaging in rent-seeking can effectively reduce competition, which may either stifle innovation through reduced competitive pressure or, conversely, incentivize innovation by securing higher returns for successful R&D investments.

A notable contribution of their study lies in its focus on the interplay between rent-seeking and market dynamics. Brou and Ruta (2007) illustrate that the extent to which rent-seeking affects innovation and market structure depends critically on the type and scale of regulatory advantages sought by firms. For example, policies that increase fixed costs disproportionately deter smaller or less efficient entrants, reshaping the competitive landscape and potentially fostering innovation among the remaining players.

However, their analysis does not delve deeply into the role of political incentives in determining rent-seeking outcomes. While they emphasize the economic motivations of firms lobbying for favorable regulations, the influence of policymakers' objectives—such as maximizing political support, campaign contributions, or personal gain—remains unexplored. This omission leaves open questions about how the interaction between firms and policymakers shapes the broader implications of rent-seeking on growth and welfare, an area that subsequent research, including Júlio (2014), begins to address.

Júlio (2014) represents a significant advancement in the study of rent-seeking by integrating political economy considerations into a general equilibrium framework. The paper builds on earlier models of rent-seeking and R&D-driven growth, introducing political motivations as a central driver of market outcomes. Júlio's model allows for interactions between economic and political markets, where firms influence policymakers to secure regulatory advantages.

In the economic market, an endogenously determined number of oligopolistic firms produce differentiated goods and engage in in-house R&D to enhance product quality. These R&D activities are crucial for sustaining innovation-driven growth, as firms invest resources to maintain a competitive edge in the market. Importantly, the model links market structure to R&D incentives, illustrating how the number and behavior of firms shape innovation outcomes.

In the political market, the same firms offer contributions to an office-motivated policymaker in exchange for favorable regulations, such as R&D licenses. These licenses act as barriers to entry, limiting competition and potentially increasing profitability for incumbents. The policymaker, motivated by a combination of political support and contributions, sets policies that balance these objectives. This interplay between firms and policymakers introduces a political dimension to market regulation, distinguishing Júlio's work from traditional rent-seeking models.

Júlio demonstrates how lobbying activities shape market concentration. By increasing entry costs through R&D licenses, lobbying can reduce the number of firms in the market. This effect has ambiguous implications for innovation: while reduced competition can stifle innovation incentives, the higher profits for incumbents may also increase R&D investments.

The model illustrates that lobbying can, under specific conditions, enhance growth and welfare. This counterintuitive result arises because lobbying, like patent protection, incentivizes R&D by ensuring higher returns for innovators. However, Júlio highlights that these benefits depend on the efficiency of political contributions and the real costs of lobbying. Welfare gains are observed only when lobbying expenditures remain below a critical threshold (e.g., 30% of the political surplus).

Júlio emphasizes the dual role of lobbying as both a growth driver and a potential market distortion. The analysis suggests that policymakers must carefully regulate lobbying to maximize its benefits while minimizing its costs, particularly in R&D-intensive industries.

While Júlio (2014) provides valuable insights into the effects of rent-seeking on innovation and growth, they leave open certain important questions. Notably, the interaction between different policymaker regimes and welfare components—consumer surplus and producer surplus—remains underexplored. Addressing these gaps, our study integrates two distinct branches of literature within the realm of political economy and firm-entry: R&D-based endogenous growth, which explores the relationship between market structure and innovation, and the political economy of rent-seeking.

For the formulation of R&D dynamics, this work draws heavily on the framework developed by Peretto (1996), which emphasizes the interplay between innovation, market concentration, and firm behavior. In addressing the political economy of rent-seeking, this study builds on Júlio (2014), who examines a general equilibrium model of R&D-driven growth, where firms engage in lobbying by offering contributions to an office-motivated policymaker in exchange for profit-enhancing regulations.

Our study diverges from Júlio (2014) in several significant ways. Júlio (2014) primarily examines two regimes—the *laissez-faire* regime and the office-motivated policymaker regime—while mentioning the benevolent policymaker regime only in passing. In contrast, our work rigorously incorporates the benevolent policymaker regime, analyzing its implications for market structure, economic growth, and welfare in detail. This expanded perspective enables a more comprehensive evaluation of how different policymaker regimes affect competition, growth, and welfare, offering actionable implications for policymaking in R&D-intensive industries.

Secondly, Júlio (2014) defines welfare as the utility of the representative household, which ties welfare directly to household consumption. In contrast, our model employs a more conventional economic definition of welfare, characterized as the sum of aggregate consumer surplus and producer surplus. This broader perspective provides a clearer distinction between consumer and producer interests, offering a more granular understanding of the trade-offs involved in different policymaker regimes. Further, unlike Júlio (2014), where households are assumed to own firms, our model treats firms and households as distinct agents. This distinction aligns with our welfare definition. By decoupling the ownership structure, our framework enables a more precise analysis of how policies impact aggregate welfare components, such as consumer surplus and producer surplus.

Similar to Júlio (2014), we do not explicitly model the process through which firms form lobbies to influence policymaker decisions. However, by focusing on the outcomes of these lobbying activities across different regimes, we provide insights into how political dynamics interact with economic growth and market structure.

We present two sets of results, focusing on market structure and the impact of market structure on economic growth and aggregate welfare. The first set of results pertains to the market structure under different types of policymaker regimes. Under the purely benevolent policymaker regime, market concentration is minimized, as the policymaker prioritizes both consumer and producer surplus equally. The free entry and exit, *laissez-faire* regime, however, results in higher market concentration due to the zero-profit condition, which naturally limits firm entry. The politically motivated policymaker regime exhibits a nuanced relationship with market structure. When a moderate weight is assigned to industry profits, the number of firms in the politically motivated regime lies between the purely benevolent and *laissez-faire* regimes. However, as the weight increases beyond a critical threshold, the politically motivated policymaker reduces the number of firms to the minimum sustainable threshold, prioritizing industry profits over competition.

The second set of results examines the relationship between market structure and its effects on economic growth and aggregate welfare. While consumer surplus increases with the number of firms, producer surplus behaves inversely, leading to an ambiguous impact on aggregate welfare. Interestingly, the highest growth rate is observed under the politically motivated policymaker regime when the weight on industry profits is moderate to strong, as reduced competition incentivizes R&D investment. Welfare, however, is maximized when the policymaker strikes a balance between consumer and producer interests. In particular, with moderate to strong weights on industry profits, welfare is highest in the politically motivated policymaker regime. These insights emphasize the importance of tailored policymaking to optimize both growth and welfare outcomes.

The rest of the paper is structured as follows. In Section 2, we present the basics of our model wherein we set up the preferences and consumer behavior, and the production and R&D technologies of firms. We then proceed to show how the goods markets clear and set up the symmetric equilibrium. In Section 3, we define the growth rate of the economy and analyze its behavior with respect to the number of firms in the economy. In Section 4, we define the aggregate welfare of the economy as a sum of the aggregate consumer surplus and the total production surplus. We then proceed to analyze how aggregate welfare behaves with respect to the total number of firms in the economy. In Section 5, we analyze the benchmark case, which is the free entry and exit, *laissez-faire* regime. We first present the industry equilibrium in this free entry and exit regime and then set up and solve for the general equilibrium. We then analyze the equilibrium growth and welfare under this regime. We use this case as a reference point for analyzing the growth and welfare implications of the other two regimes in our model.

In Section 6, we lay out the purely benevolent policymaker regime, wherein the policymaker determines the market structure by maximizing the aggregate welfare of the economy.

We then proceed to determine the industry equilibrium under this regime. In Section 7, we set up the politically motivated policymaker regime, wherein the policymaker determines the market structure by maximizing the sum of aggregate consumer surplus and total producer surplus, with a higher weight on the latter. We then characterize the industry equilibrium under this regime.

Section 8 characterizes the general equilibrium in the purely benevolent and politically motivated policymaker regimes. In Section 9, we analyze the growth and welfare implication of the two regimes in which the policymaker determines the market structure, vis-à-vis the *laissez-faire* regime. Due to mathematical intractability, we resort to numerical calibration exercises to make these comparisons in our present work. We present the main results of our paper in Section 10 and conclude in Section 11.

2 The Model

In this study, we develop an analytical general equilibrium model within a dynamic framework, incorporating endogenous R&D and rent-seeking behaviors. The model operates in continuous time. The economy consists of a unit mass of identical, infinitely-lived households. Each household derives utility from consuming differentiated goods supplied by firms and supplies one unit of labor inelastically. Full employment is assumed.

There are $N > 1$ oligopolistic firms operating in the economy, each producing a unique variety of a differentiated good using the existing technology[1]. The number of firms itself is determined differently in each of the three different alternative regimes in which the economy could operate. The first is the free entry and exit *(laissez-faire)* regime, where the number of firms is determined by the zero profit condition. Thus, in this case, N is determined by market forces.

The other two regimes involve a policymaker who regulates—or endogenously sets—the number of firms N in the economy by formulating a firm-entry policy. The policymaker is

[1] Since N is the number of firms, the main analysis of our paper considers it as a discrete variable. However, in certain steps, we consider N to be a continuous variable, so as to simplify the algebra of our model.

taken to have exclusive authority over the issuance of R&D licenses, granting him/ her the ability to regulate the number of firms N in the economy by facilitating or restricting entry. Building on this framework, the second regime in our model is the benevolent policymaker regime, where the policymaker determines the number of firms, N, by maximizing pure aggregate welfare, defined as the sum of aggregate consumer surplus and aggregate producer surplus.

In the third regime, the policymaker is politically motivated, and he/ she strives to retain office. This means that he/ she not only needs to maximize pure social welfare but also get political support from firms, which is needed to run for elections[2]. These incentives align the policymaker's objectives with those of industry lobbies, potentially leading to decisions that favor market concentration. Therefore, under this regime, the policymaker sets the number of firms, N, by maximizing the sum of aggregate consumer surplus and aggregate producer surplus with a higher weight on the latter.

Firms in this model undertake R&D investments to improve their state-of-the-art product, which enhance consumer utility. However, the model does not follow any Schumpeterian features of creative destruction, where new firms displace incumbents. Instead, entry increases the variety of goods available in the economy, while R&D activities improve the quality of existing products.

The interaction between the agents in the economy is structured as follows: Households purchase goods from firms, incurring expenditure E, and receive wages, w in return for their labor. Firms generate profits, Π, which are partly used to provide political support (e.g., campaign contributions) to the policymaker. The policymaker, in turn, defines the market structure by regulating the number of firms, N. However, we do not explicitly model how this support or these contributions accrue to the policymaker. It is implicit in our analysis that a share of the profits made by all the firms in the economy is offered to the policymaker in exchange for setting a firm-entry policy that would be favorable to them.

The rest of this section systematically develops the core components of the model, starting with consumer preferences and behavior, followed by the production and R&D technologies

[2]Political support from the firm- or industry-lobbies may take the form of bribes or campaign contributions, but these have not been modelled explicitly in our work.

of firms, and finally, the determination of equilibrium in goods and labor markets.

2.1 Consumer Preferences

We begin by setting up the preference structure and behavior of consumers in our model. The analysis adopts a representative agent framework, where individuals are identical and infinitely lived. A representative household maximizes its lifetime utility,

$$u(t) = \int_t^\infty log(C(\tau)) \cdot e^{-(\rho(\tau-t))} d\tau,$$

where $C(\tau)$ is the level of consumption at time τ, and $\rho > 0$ is the discount factor, which reflects the degree to which individuals value current consumption over future consumption. The utility function assumes logarithmic preferences, implying diminishing marginal utility from consumption over time.

Households face the following lifetime intertemporal budget constraint

$$\int_t^\infty E(\tau) \cdot e^{-R(\tau)} d\tau \leq \int_t^\infty w(\tau) \cdot e^{-R(\tau)} d\tau,$$

where $E(\tau)$ is the household's expenditure, $w(\tau)$ is the wage earned by the household, and $\rho > 0$ is the discount factor. $R(\tau) = \int_t^\tau r(s)ds$ is the accumulated average interest rate between time period t and τ.

Let P_C denote the price index of consumption goods. Then, household expenditure and consumption are linked as $E = P_C \cdot C$, where C is the quantity of consumption. Solving the intertemporal utility maximization problem yields the growth rate of expenditure over time, $\dot{E}/E = r - \rho$.

Consumers derive utility from consuming differentiated goods, x_i, each characterized by a state-of-the-art quality index, q_i. The consumption bundle is specified following the framework of Dixit and Stiglitz (1977), as

$$C = \left[\sum_{i=1}^{N} (q_i \cdot x_i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where $\varepsilon > 1$ is the elasticity of substitution between different varieties. This formulation captures consumers' preference for variety. A higher ε implies goods are closer substitutes, while a lower ε indicates stronger differentiation between varieties.

Households allocate their expenditure across the N goods by maximizing the utility function subject to the budget constraint

$$E = \sum_{i=1}^{N} p_i \cdot x_i,$$

where p_i is the price of good i.

The consumer maximization problem can be expressed as

$$\mathcal{L} = \int_t^{\infty} \left[\log \left(\left[\sum_{i=1}^{N} (q_i x_i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \right) e^{-\rho(\tau-t)} - \lambda(\tau) \left(\sum_{i=1}^{N} p_i x_i - E(\tau) \right) \right] d\tau,$$

where λ is the Lagrange multiplier, representing the shadow price of expenditure. The first-order condition would be

$$\frac{1}{\left[\sum_{i=1}^{N} (q_i x_i)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}} \cdot \frac{\varepsilon}{\varepsilon-1} \cdot \left[\sum_{i=1}^{N} (q_i x_i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{1}{\varepsilon-1}} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot (q_i x_i)^{-1/\varepsilon} q_i = \lambda p_i.$$

This maximization exercise yields

$$x_i\left(p_i, q_i\right) = \frac{E \cdot S_i\left(p_i, q_i\right)}{p_i},$$

where

$$S_i\left(p_i, q_i\right) = \frac{p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}}$$

represents the market share captured by the firm i. This term reflects the relative importance of price and quality in determining a firm's share of household expenditure.

[Refer to Appendix 1 for a detailed derivation.]

We begin by normalizing the initial or base quality level of each variety of goods to unity, such that $q_i(t) = 1, \ \forall i$. Since consumers are identical and the population in the economy is normalized to one, the aggregate demand for a good, i, is equivalent to the demand of the representative consumer. Thus, the demand function for any variety, i, can be expressed as

$$X^D\left(p_i, q_i\right) = \frac{E \cdot S_i\left(p_i, q_i\right)}{p_i}.$$

To analyze consumer behavior further, we compute two key elasticities: the price elasticity of demand and the quality elasticity of demand. Accordingly, the price elasticity of demand will be

$$\xi\left(p_i, q_i\right) = -\frac{dX_i^D}{dp_i} \cdot \frac{p_i}{X_i^D} = \varepsilon - (\varepsilon - 1) \cdot S_i\left(p_i, q_i\right),$$

where ε captures the consumer's propensity to substitute between varieties of goods. The equation indicates how the quantity demanded responds to a change in price, adjusted by the market share of the good, $S(p_i, q_i)$. Specifically, goods with higher market share experience smaller changes in demand relative to price changes, as they are more entrenched in consumer preferences.

The quality elasticity of demand will be

$$\zeta(p_i, q_i) = \frac{dX_i^D}{dq_i} \cdot \frac{q_i}{X_i^D} = (\varepsilon - 1)[1 - S_i(p_i, q_i)],$$

which shows how the quantity demanded changes in response to the quality of the good, moderated by its market share, $S(p_i, q_i)$. The adjustment factor, $[1 - S_i(p_i, q_i)]$, shows that goods with smaller market shares exhibit a stronger responsiveness to quality changes. Intuitively, consumers are more sensitive to quality improvements for less entrenched varieties in the market.

2.2 Production Technology, R&D and Firm Behavior

We now describe the production side of this economy, including the interaction between production processes, R&D investments, and firm behavior. Each firm produces a differentiated product using a linear production technology, specified as

$$L_{X_i} = X_i + \phi, \tag{2}$$

where X_i is the output produced by firm i and sold to households, and L_{X_i} is the total labor employed by firm i in production. The parameter $\phi > 0$ represents the fixed (and sunk) cost of production, interpreted as the labor allocated to overhead or administrative activities. This cost is independent of the scale of production. The production technology implies that firms incur fixed costs regardless of their production level, while variable costs depend linearly on the amount of output produced.

While production relies on labor and fixed costs, product quality improvements—essential for maintaining competitiveness—are driven by R&D investments. Each firm allocates labor resources to R&D to enhance its knowledge stock, z_i, which directly influences the quality of its product. The firm's quality stock, q_i, determines the quality embedded in the state-of-the-art product, and is directly related to the firm's knowledge z_i by the postulation $q_i = z_i^{\theta}$, where θ is the elasticity of quality with respect to R&D investment. A higher θ implies larger quality improvements for a given R&D investment, incentivizing firms to allocate more resources to R&D when quality improvements have a high payoff.

In our model, we do not capture the Schumpeterian "creative destruction" effect, and hence, the R&D investment decision undertaken by firms is independent of any entry threat faced by the existing firms. As new firms enter, they do not displace the existing firms but only add yet another variety of product into the market, and their R&D effort improves the quality of their product over time. The parameter z_i is assumed to evolve according to

$$\dot{z}_i = L_{z_i} \cdot \left[z_i + \gamma \sum_{j \neq i}^{N} z_j \right] = L_{z_i} \cdot Z_i, \tag{3}$$

where $\dot{z}_i = dz_t/dt$ is the number of new patents produced in $d\tau$ units of time by a firm employing L_{z_i} units of labor in R&D. This is similar to Peretto (1996) and Júlio (2014). The term $Z_i = \left[z_i + \gamma \sum_{j \neq i}^{N} z_j \right]$ is the total productivity of R&D investment made by firm i. The productivity in the R&D sector is a linear combination of both private (excludable) and public (non-excludable) knowledge, with $\gamma \in (0, 1)$ determining the share of private research that becomes publicly available. Thus, γ is the R&D spillover. Higher spillovers reduce the exclusivity of private R&D but also enhance the collective productivity of the industry, potentially balancing competition and innovation.

Note that $\gamma = 0$ would mean that the competitor's knowledge is a private good and $\gamma = 1$ would mean that it is a pure public good. In keeping with quality-ladders models like Grossman and Helpman (1991), the spillover of competitor's knowledge can be interpreted to mean that when an innovator comes up with a new product in the market, other researchers can disassemble it and study its attributes, which thereby helps them to develop new blueprints and increase their own R&D productivity. The evolution of knowledge as formulated in Equation (3) exhibits constant returns to scale in knowledge and overall increasing returns to scale[3].

[3] To demonstrate constant returns to scale (CRS) in knowledge, we scale the knowledge inputs z_i and z_j

The typical firm maximizes the following net present value of cash flows

$$V_i(t) = \int_t^{\infty} \pi_i(\tau) \cdot e^{-R(\tau)} \, d\tau, \tag{4}$$

where V_i is the perfect-foresight stock market value of the firm, that is, the price of the share owned by an equity holder. The instantaneous profits are given by

$$\pi_i = p_i \cdot X^D \left(p_i, q_i\right) - w \cdot \left(L_{X_i} + L_{z_i}\right).$$

For our analysis, through appropriate choice of units, we set the wage rate to be equal to 1 (that is, $w = 1$), since we measure all variables in terms of the wage rate. Therefore, the instantaneous profits are now given by

$$\pi_i = p_i \cdot X^D \left(p_i, q_i\right) - \left(L_{X_i} + L_{z_i}\right). \tag{5}$$

Profit maximization works through the choice of a price strategy, p_i, and an R&D strategy, L_{z_i}, subject to the technological constraints given by Equation (2) and Equation (3), and total demand $X^D \left(p_i, q_i\right)$. For this, the number of firms, and the competitors' pricing strategies and R&D investments are taken as given.

Following Peretto (1996), we consider a symmetric Nash equilibrium in open loop strategies. Let $a_i = [p_i(\tau), L_{z_i}(\tau)]$ for $\tau \geq t$ be firm i's strategy vector. These strategies are time-paths of prices and R&D investments, the latter determined by labor input in the R&D

by a factor λ. The scaled equation becomes

$$\dot{z_i}' = L_{zi}\Big[\lambda z_i + \gamma \sum_{j\neq i} \lambda z_j\Big] = L_{zi}\lambda\Big[z_i + \gamma \sum_{j\neq i} z_j\Big] = \lambda L_{zi} Z_i.$$

This shows that scaling the knowledge inputs by λ results in the output being scaled by λ, confirming CRS. Similarly, to check for overall increasing returns to scale, we scale both the knowledge inputs and the labor by λ. The scaled equation becomes

$$\dot{z_i}'' = \lambda L_{zi}\Big[\lambda z_i + \gamma \sum_{j\neq i} \lambda z_j\Big] = \lambda^2 L_{zi}\Big[z_i + \gamma \sum_{j\neq i} z_j\Big] = \lambda^2 L_{zi} Z_i.$$

Since scaling all the inputs by λ results in the output being scaled by λ^2, this indicates that the production function exhibits increasing returns to scale overall. This occurs because the positive externalities from spillovers enhance the productivity of the combined R&D efforts, leading to a more than proportional increase in output.

sector. To simplify the analysis, we assume that entry entails zero costs and firms do not have any scrap value. Thus, the number of firms is free to jump to its equilibrium level. At time t (the initial time period), firms commit to a time-path strategy in prices and R&D investment. Simultaneously, free entry and exit or the benevolent or office-motivated planner determine the equilibrium number of firms in the market, as the case may be (these individual cases are discussed in separate sections later on). These price and investment strategies played by the firms induce time-paths of production, sale and knowledge accumulation. Thus, at time t the vector $[N, a_1, a_2, \ldots a_N]$ is an instantaneous equilibrium if

$$V_i[N, a_1, a_2, \ldots, a_i, \ldots, a_N] \geq V_i[N, a_1, a_2, \ldots, a_i', \ldots, a_N] \geq 0, \forall i \tag{A}$$

and

$$V_i[N+1, a_1, a_2, \ldots, a_i, \ldots, a_{N+1}] \leq 0, \forall N > 1 \tag{B}$$

where $V_i[N, a_1, a_2, \ldots, a_i', \ldots, a_N]$ is firm i's strategy vector when it deviates from its optimal time-paths of prices and investment in R&D while all the other firms do not deviate. Equation (A) ensures firms optimize strategies and remain profitable. Equation (B) ensures no excessive entry, maintaining a stable equilibrium. In Equation (A), the first inequality means that the firm maximizes the present value of net cash flow, taking as given the strategies of all the other competitor firms. The second inequality means that this maximized value should be nonnegative. This second inequality is also the free-exit condition. Equation (B) is the standard free entry condition that the value of the marginal firm be nonpositive. Thus, we analyze a one-shot game played at time t, which defines the future behavior of all variables in the economy.

2.3 The Goods Market

In this section, we solve for the goods market equilibrium by determining the firm's optimal price and R&D investment strategies. Firms aim to maximize their profit stream, subject to their production technology, R&D productivity, and the demand for differentiated goods. The firm's decision-making is framed as a dynamic optimization problem, where R&D investments affect future knowledge capital and product quality. To solve this optimization problem, we formulate the current-value Hamiltonian as

$$H_i^{cv} = p_i \cdot X^D(p_1, q_i) - (L_{x_i} + L_{x_i}) + \mu_i \cdot L_{z_i} \left[z_i + \gamma \sum_{j \neq i}^{N} z_j \right].$$

Substituting for $X^D(p_i, q_i)$ and L_{x_i}, we have,

$$H_i^{cv} = p_i \cdot \frac{ES_i(p_i, q_i)}{p_i} - \left[\frac{ES_i(p_i, q_i)}{p_i} + \phi + L_{z_i} \right] + \mu_i \cdot L_{z_i} \left[z_i + \gamma \sum_{j \neq i}^{N} z_j \right],$$

which simplifies to

$$H_i^{cv} = (p_i - 1) \cdot \frac{ES_i(p_i, q_i)}{p_i} - (L_{z_i} + \phi) + \mu_i \cdot L_{z_i} \left[z_i + \gamma \sum_{j \neq i}^{N} z_j \right],$$

where the co-state variable, μ_i, is the shadow price of the knowledge capital z_i. It measures the value of the marginal unit of knowledge, *i.e.*, the value of the patent, in terms of its contribution to the firm's future profits. The state variable is the firm's knowledge capital, z_i. The control variables are R&D investment, L_{z_i}, and price, p_i.

Recall that

$$S_i\left(p_i, q_i\right) = \frac{p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}}.$$

Therefore, the Hamiltonian is now rewritten as

$$H_i^{cv} = (p_i - 1) \cdot \frac{E p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}}{p_i \sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}} - (L_{z_i} + \phi) + \mu_i \cdot L_{z_i} \left[z_i + \gamma \sum_{j \neq i}^{N} z_j \right].$$

The first order condition with respect to the price, p_i would be

$$p_i \sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)} \left[-q_i^{(\varepsilon-1)} (p_i - 1)(\varepsilon - 1) p_i^{-\varepsilon} + p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1} \right]$$
$$- (p_i - 1) p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)} \left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)} - p_i q_i^{(\varepsilon-1)} (\varepsilon - 1) p_i^{-\varepsilon} \right] = 0,$$

which, upon dividing both sides by $p_i^{-(\varepsilon-1)} \cdot q_i^{(\varepsilon-1)}$, gives

$$- \left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)} \right] \left[(\varepsilon - 1)(p_i - 1) \right] + p_i \left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)} \right]$$

$$- (p_i - 1) \left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)} - p_i^{-(\varepsilon-1) q_i^{(\varepsilon-1)}} \right] = 0.$$

This can be further simplified to

$$-\left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}\right][(\varepsilon-1)(p_i-1)] + \sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)} = -(\varepsilon-1)(p_i-1) \cdot p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}.$$

By rearrangement of terms, this can then be rewritten as

$$-(\varepsilon-1)(p_i-1)+1 = -(\varepsilon-1)(p_i-1)\frac{p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}}.$$

Substituting for

$$S_i\left(p_i, q_i\right) = \frac{p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}},$$

the first order condition in price can be rewritten as

$$-(\varepsilon-1)(p_i-1)+1 = -(\varepsilon-1)(p_i-1)S_i(p_i, q_i).$$

Collecting $(p_i - 1)$ terms on the left-hand side, this can be rewritten as

$$(p_i-1)[(\varepsilon-1) - S_i(p_i, q_i) \cdot (\varepsilon-1)] = 1,$$

which can also be solved to yield:

$$p_i = \frac{1}{(\varepsilon-1) - S_i(p_i, q_i) \cdot (\varepsilon-1)} + 1,$$

which further simplifies to

$$p_i = \frac{\varepsilon - S_i(p_i, q_i)(\varepsilon-1)}{\varepsilon - S_i(p_i, q_i)(\varepsilon-1) - 1}.$$

We know that $\xi\left(p_i, q_i\right) = \varepsilon - (\varepsilon-1)S_i\left(p_i, q_i\right)$. Therefore, the first order condition in price can now be rewritten as

$$p_i = \frac{\xi_i}{\xi_i - 1}, \tag{6}$$

where ξ_i is the price elasticity of demand, as seen previously.

[Refer to Appendix 2 for a detailed derivation.]

Equation (6) shows that the optimal price is a markup over the marginal cost, which is determined by the price elasticity of demand ξ_i. Specifically, the optimal price p_i reflects a higher markup when the elasticity of demand is low (indicating less sensitivity to price changes) and a lower markup when the elasticity is high (indicating greater sensitivity).

The price elasticity of demand itself is a function of the degree of substitutability between the varieties of goods, ε, and the market share of each firm, $S(p_i, q_i)$. This relationship underscores the degree of market power each firm holds in a monopolistic market with CES demand, where differentiated goods allow firms to set prices above marginal costs.

The first order condition with respect to the R&D investment, L_{z_i}, will be

$$\mu_i \left[z_i + \gamma \sum_{j \neq i}^{N} z_j \right] = \mu_i Z_i = 1, \tag{7}$$

with $0 < L_{z_i} < 1$. The optimal R&D strategy given by Equation (7) implies that the marginal revenue from one unit of R&D is equal to its marginal cost. Here, μ_i represents the shadow price of knowledge, and Z_i is the effective knowledge, including spillovers. This implies that firms invest in R&D up to the point where the value of the marginal knowledge generated equals the cost of R&D investment, ensuring that R&D investments are optimized for profit maximization.

The evolution of the costate variable, μ_i, is governed by

$$\frac{(p_i - 1)}{p_i} \cdot E \cdot \frac{\partial S_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial z_i} + \mu_i L_{z_i} = -\mu_i + r\mu_i.$$

Substituting for $q_i = z_i^\theta$ and for p_i from Equation (6), the above equation can be rewritten as

$$\frac{E}{\xi_i} \cdot \frac{\partial S_i}{\partial q_i} \cdot \frac{\theta q_i}{z_i} + \mu_i L_{z_i} = -\mu_i + r\mu_i.$$

Multiplying the first term on the left-hand side by $(\frac{S_i}{S_i} \cdot 1)$ and substituting for 1 from Equation (7) the above equation becomes

$$\frac{E}{\xi_i} \cdot \frac{\partial S_i}{\partial q_i} \cdot \frac{S_i}{S_i} \cdot \mu_i Z_i \cdot \frac{\theta q_i}{z_i} + \mu_i L_{z_i} = -\mu_i + r\mu_i,$$

which can be re-expressed as

$$E \cdot \frac{\theta S_i}{\xi_i} \cdot \frac{\partial S_i}{\partial q_i} \cdot \frac{q_i}{S_i} \cdot \frac{\mu_i Z_i}{z_i} + \mu_i L_{z_i} = -\mu_i + r\mu_i.$$

Here, the term $\frac{\partial S_i}{\partial q_i} \cdot \frac{q_i}{S_i}$ is the quality elasticity of demand, ζ_i, as defined earlier. Therefore, the above equation can be rewritten as

$$\theta E S_i \cdot \frac{\zeta_i}{\xi_i} \cdot \frac{\mu_i Z_i}{z_i} + \mu_i L_{z_i} = -\mu_i + r\mu_i,$$

which can rearranged to yield

$$r = \theta ES_i \cdot \frac{\zeta_i}{\xi_i} \cdot \frac{Z_i}{z_i} + L_{z_i} + \frac{\dot{\mu}_i}{\mu_i}. \tag{8}$$

Equation (8) is the no-arbitrage condition that ensures that the return on R&D investment is equal to the exogenous market interest rate on a riskless asset, r, reflecting the trade-off between investing in knowledge capital and other financial assets. It combines the effects of consumer demand, market share, and the productivity of R&D efforts, indicating that firms allocate resources efficiently between R&D and other opportunities to maximize returns.

Finally, the transversality condition is

$$\lim_{t\to\infty} \mu_i(\tau) \cdot z_i(\tau) \cdot e^{-R(\tau)} = 0.$$

This condition ensures that the value of knowledge capital converges to zero at the end of the planning horizon. It prevents firms from accumulating infinite value from knowledge, aligning long-term investment decisions with finite time horizons.

These equilibrium conditions, given by Equation (6), Equation (7), and Equation (8), and the transversality condition, collectively describe how firms determine their pricing and R&D investment strategies in a dynamic, monopolistic market with differentiated goods. They highlight the interplay between market power, consumer demand, and the productivity of R&D investments, setting the stage for analyzing the effects of different market structures on growth and welfare in subsequent sections.

The equilibrium conditions derived here—equations for optimal price, R&D investment, the costate variable, and the transversality condition—together define the firm's strategy in a dynamic setup. Key insights that emerge from the optimization problem are as follows. Firms exploit market power by setting prices as a markup over marginal cost, influenced by demand elasticity and market share. In the R&D sector, they balance private and spillover effects in R&D, investing until the marginal revenue equals the marginal cost. The costate equation links R&D productivity, demand elasticities, and the shadow price of knowledge, ensuring efficient allocation of resources over time.

2.4 The Symmetric Equilibrium

We now derive equilibrium conditions under the assumption of symmetry across firms. All firms face the same demand function and have the same production technology and cost structure, as well as the R&D technologies. Labor is also homogeneous. Further, knowledge spillovers create uniform effects across firms, allowing us to assume that all firms reach the same equilibrium in R&D investment. Therefore, we restrict ourselves to the symmetric equilibrium so as to focus on the growth path and aggregate welfare of the economy. Unless explicitly stated, variables without subscripts are considered industry averages, consistent with the approach in Peretto (1996).

It is important to note that firms are distinguished by the quality of their product, q_i. Since firms have the same R&D technology and labor is homogeneous, the effort required to bring about a quality improvement in an existing product is the same across all firms, but the quality embedded in each of the products is by itself different. Consumers perceive this differentiated quality of each variety as such.

Recall that $q_i = z_i^{\theta}$. We take the natural logarithm and differentiate with respect to time

$$\frac{\dot{q}}{q} = \theta \frac{\dot{z}}{z}. \tag{9}$$

Equation (9) tells us that the evolution of quality stock over time $(\dot{q}/q)$, is directly proportional to the evolution of knowledge stock over time $(\dot{z}/z)$, and the productivity of R&D sector, θ.

We know from Equation (3) that

$$\dot{z} = L_z \cdot \left[z + \gamma \sum_{j \neq i}^{N} z \right], \tag{10}$$

where, on dividing both sides by z, we have

$$\frac{\dot{z}}{z} = L_z \left[1 + \gamma(N-1) \right]. \tag{11}$$

Substituting Equation (11) in Equation (9), we get

$$\frac{\dot{q}}{q} = \theta \frac{\dot{z}}{z} = \theta \left[1 + \gamma(N-1) \right] L_z = \theta \sigma(N) L_z. \tag{12}$$

Equation (12) describes the evolution of quality stock over time as a function of R&D investment by the firm, represented by the elasticity of quality with respect to R&D investment, θ, the average labor employed in the R&D sector, L_z, and the term $\sigma(N) = [1 + \gamma(N-1)]$, which represents the productivity of an R&D project by applying one unit of labor. Notice that $\sigma(N)$ is increasing in N, which reflects a positive R&D externality. Since the free-entry condition determines the equilibrium number of firms at each moment in time, profits are instantaneously eliminated by the costless entry/exit of firms. Equation (12) reveals that while firms are distinguished by their quality, the rate of quality evolution itself is the same across all firms.

In Equation (3) we have defined $Z = \left[z_i + \gamma \sum_{j\neq 1}^{N}\right]$, which can now be rewritten as $\sigma(N) \cdot z$, and hence we have

$$Z = \sigma(N) \cdot z. \tag{13}$$

Taking the natural logarithm of both sides and differentiating with respect to time yields

$$\frac{\dot{Z}}{Z} = \frac{\dot{z}}{z}. \tag{14}$$

In the symmetric equilibrium, Equation (7) now becomes

$$\mu Z = 1. \tag{15}$$

Equation (15) describes the optimal relationship between the shadow price of knowledge, μ, and the total productivity of the R&D sector, Z. Here, μ indicates the value of additional knowledge in terms of its contribution to the firm's value. Z captures the overall effectiveness of R&D efforts, including both private and public knowledge components. This equation implies that firms invest in R&D until the marginal revenue from R&D investment equals the marginal cost, ensuring an optimal balance in their R&D strategies within the context of a symmetric equilibrium.

Differentiating Equation (15) with respect to time, yields

$$\frac{\dot{\mu}}{\mu} = -\frac{\dot{Z}}{Z}. \tag{16}$$

Equation (16) establishes a dynamic relationship between the growth rate of the shadow price of knowledge, $\frac{\dot{\mu}}{\mu}$, and the growth rate of the effective knowledge stock, $\frac{\dot{Z}}{Z}$. The negative proportionality between these rates of change implies that as the knowledge stock increases, its marginal value decreases, guiding firms to adjust their R&D investments to maintain an optimal balance between the marginal cost and benefit of additional knowledge.

Substituting Equation (12) and Equation (16) in Equation (8), and using the fact that in a symmetric equilibrium $\frac{Z}{z} = \sigma(N)$ and $S = 1/N$, we obtain

$$r = \frac{E}{N\xi} \cdot \theta\zeta\sigma(N) + L_z - \frac{\dot{Z}}{Z}. \tag{17}$$

Equation (17) describes the rate of return on R&D investment, r, in the symmetric equilibrium. Here, the term $\frac{E}{N\xi}$ represents the effective expenditure per firm, which is the total expenditure, E, adjusted for the number of firms, N, and the price elasticity of demand, ξ. The term $\theta\zeta\sigma(N)$ captures the productivity of R&D efforts, with θ being the elasticity of quality with respect to R&D investment, ζ being the quality elasticity of demand, and $\sigma(N)$ being the productivity function reflecting R&D spillovers. L_z represents the optimal level of R&D investment, which ensures that the marginal revenue from one unit of R&D is equal to its marginal cost, balancing the investment in innovation. The term $-\frac{\dot{Z}}{Z}$ adjusts for the growth rate of the knowledge stock, indicating diminishing returns on additional R&D investment as the market becomes saturated with knowledge. This term reflects how quickly the overall knowledge in the economy is growing, influencing the firm's decision on how much to invest in R&D.

Equation (17) states that a higher expenditure per firm and greater productivity of R&D efforts lead to a higher rate of return. However, the term $-\frac{\dot{Z}}{Z}$ adjusts the rate of return downward to account for the diminishing returns as the knowledge stock grows, indicating that as the market becomes more saturated with knowledge, the returns on additional R&D investment decrease.

From Equation (12) we have $\sigma(N) = [1+\gamma(N-1)]$ and from Equation (12) and Equation (14) we have $\frac{\dot{Z}}{Z} = \frac{\dot{z}}{z} = \sigma(N)L_z$. Substituting these in Equation (17), we have

$$r = \frac{E}{N\xi} \cdot \theta\zeta[1+\gamma(N-1)] + L_z - [1+\gamma(N-1)]L_z, \tag{18}$$

which simplifies to

$$r = \frac{E}{N\xi} \cdot \theta\zeta[1+\gamma(N-1)] - \gamma(N-1)L_z, \tag{19}$$

where the terms $\xi = \varepsilon - \frac{(\varepsilon-1)}{N}$ and $\zeta = \frac{(\varepsilon-1)\cdot(N-1)}{N}$ are the price and quality elasticities of demand, respectively.

Equation (19), which is the no-arbitrage condition in the context of a symmetric equilibrium, helps us identify the determinants of average R&D investment and, by extension, economic growth. The term $\frac{E}{N\xi}$ denotes the *gross-profit effect*, representing the gross profit of a firm for a given market share. The term $\theta\zeta$ is known as the *business-stealing effect*, which reflects the increase in market share driven by quality-enhancing R&D. Additionally, *technology spillovers* also have two distinct sub-effects over R&D productivity. The first sub-effect is represented by the term $-\gamma(N-1)$, which indicates that firms recognize that their own R&D generates spillovers, which makes their competitors more productive. The second sub-effect, represented by the positive term $\gamma(N-1)$, signifies the benefits firms gain from the spillovers of other firms, which enhances their own productivity.

These effects will be crucial in the subsequent analysis to understand the dynamics of R&D investment decisions and their impact on economic growth. Specifically, the gross-profit effect will help us assess the profitability and sustainability of firms' R&D investments. The business-stealing effect will be used to analyze competitive strategies and how firms leverage R&D to increase their market share. Lastly, the technology spillovers will be pivotal in understanding the overall productivity gains in the economy and the interdependence of firms' R&D activities. By examining these effects in detail, we will be able to derive comprehensive insights into the mechanisms driving economic growth and innovation.

Equation (19) can be rewritten as

$$\gamma(N-1)L_z = \frac{E}{N\xi} \cdot \theta\zeta[1 + \gamma(N-1)] - r,$$

which simplifies to

$$L_z(N, E, r) = \frac{1}{\gamma}\left[\theta \cdot \zeta(N)\frac{E}{N\xi(N)} \cdot \frac{\sigma(N)}{N-1} - \frac{r}{N-1}\right]. \tag{20}$$

Equation (20) gives the optimal individual investment in R&D as a function of the number of firms, N, aggregate demand, E, and the interest rate, r, for an interior solution. Note that the optimal investment in R&D given by Equation (20) directly follows from the no-arbitrage condition in the symmetric equilibrium case, which was given by Equation (19).

2.4.1 Relationship Between the Average Investment in R&D and the Number of Firms, N

From Equation (20), we analyze the shape of the average R&D investment, L_z, and the aggregate R&D investment, $\mathbf{L}_z$, with respect to the number of firms, N. We will use these results in the next section when we analyze the relationship between the growth rate of the economy and the number of firms.

We begin with examining the relationship between the average R&D, L_z and the number of firms, N, which is expressed as the following lemma.

Lemma 1 ***The average investment in R&D,*** L_z***, exhibits a hump-shaped pattern with respect to the number of firms,*** N***.***

Proof. The formal proof that L_z exhibits a hump-shaped pattern with respect to N involves three steps. First, we demonstrate that $\partial L_z/\partial N > 0$ for small values of N, indicating that L_z is increasing. Second, we establish that $\lim_{N\to\infty} \partial L_z/\partial N$ converges to zero, suggesting that L_z reaches a peak and levels off. Finally, we show that $\partial L_z/\partial N < 0$ for large values of N, confirming that L_z decreases after the peak.

From Equation (20), we know that

$$L_z(N, E, r) = \frac{1}{\gamma}\left[\theta \cdot \zeta(N)\frac{E}{N\xi(N)} \cdot \frac{\sigma(N)}{N-1} - \frac{r}{N-1}\right].$$

Substituting for the price elasticity of demand, $\xi(N) = \varepsilon - \frac{(\varepsilon-1)}{N}$, quality elasticity of demand, $\zeta(N) = \frac{(\varepsilon-1)(N-1)}{N}$, and $\sigma = [1+\gamma(N-1)]$ in the above equation, we have

$$L_z(N, E, r) = \frac{1}{\gamma}\left[\frac{E\theta}{N} \cdot \frac{(\varepsilon-1)(N-1)}{N\varepsilon - (\varepsilon-1)} \cdot \frac{[1+\gamma(N-1)]}{N-1} - \frac{r}{N-1}\right].$$

The partial derivative of L_z, with respect to N, would be

$$\frac{\partial L_z}{\partial N} = \frac{1}{\gamma} \cdot \frac{\theta(\varepsilon-1)E}{N(N\varepsilon-(\varepsilon-1))}\left[\gamma - \frac{(1+\gamma(N-1))(2N\varepsilon-(\varepsilon-1))}{N(N\varepsilon-(\varepsilon-1))}\right] + \frac{r}{\gamma}\frac{1}{(N-1)^2}.$$

For small N, that is, as N approaches 1, we have,

$$\left.\frac{\partial L_z}{\partial N}\right|_{(N\to 1)} = \frac{1}{\gamma}\theta(\varepsilon-1)E(\gamma-\varepsilon-1) + \frac{r}{\gamma}\frac{1}{\lim_{N\to 1}(N-1)^2} = +\infty.$$

Similarly,

$$\left.\frac{\partial L_z}{\partial N}\right|_{(N\to\infty)} = \frac{\theta(\varepsilon-1)E}{\lim_{N\to\infty} N(N\varepsilon-(\varepsilon-1))} + \frac{r}{\gamma}\frac{1}{\lim_{N\to\infty}(N-1)^2} = 0.$$

Showing that $\partial L_z/\partial N < 0$ for large N, is equivalent to showing that

$$\frac{\theta(\varepsilon-1)E(N-1)}{N(N\varepsilon-(\varepsilon-1))}\left[\frac{(1+\gamma(N-1))(2N\varepsilon-(\varepsilon-1))}{N(N\varepsilon-(\varepsilon-1))} - \gamma\right] > \frac{r}{N-1}.$$

The left-hand side of the above the equation simplifies to

$$\frac{\theta(\varepsilon-1)E}{N(N\varepsilon-(\varepsilon-1))}\left[(1+\gamma(N-1))\frac{(N-1)}{N} + \frac{(1+\gamma(N-1))\varepsilon(N-1)}{N\varepsilon-(\varepsilon-1)} - \gamma(N-1)\right].$$

For large N, the above equation converges to

$$\frac{\theta(\varepsilon-1)E}{N(N\varepsilon-(\varepsilon-1))}(2+\gamma(N-1)) > \frac{r}{N-1},$$

where the inequality arises for any positive non-zero investment in R&D, $L_z > 0$. Thus, the average R&D investment, L_z is hump-shaped.

[Refer to Appendix 3 for a detailed derivation of the steps.]

The hump-shaped relationship between L_z and N arises from two competing effects: the *gross-profit effect* and the *business-stealing effect*. The *gross-profit effect* suggests that returns to R&D decrease as N increases because a larger number of firms reduces both market share and markup for each firm. This results in lower profits and, consequently, diminished incentives to invest in quality improvements. Conversely, the *business-stealing effect* implies that firms are more inclined to invest as N rises since the potential market share gains from R&D are greater. The *business-stealing effect* prevails when the number of firms is small, as the total market profits that can be captured through R&D are higher. However, the *gross-profit effect* becomes dominant when N is large, as the profits obtainable through quality enhancements decrease. These dynamics hold true regardless of the R&D spillover effects.

2.4.2 Relationship Between the Aggregate R&D Investment and the Number of Firms, N

We now proceed to analyze the relationship between the aggregate R&D investment, $\mathbf{L}_z = NL_z$, and the number of firms, N. We express the relationship as the following lemma.

Lemma 2 ***The aggregate R&D investment, L_z is hump-shaped in the number of firms, N, if and only if the interest rate is sufficiently low.***

Proof. The formal proof that aggregate R&D investment, $\mathbf{L}_z$, is hump-shaped with respect to the number of firms involves three steps. We first show that the first derivative of aggregate R&D with respect to the number of firms, $\frac{\partial \mathbf{L}_z}{\partial N}$, is positive for small values of N. This demonstrates that aggregate R&D initially increases as the number of firms increases from a small value. Next, we establish that the first derivative, $\frac{\partial \mathbf{L}_z}{\partial N}$, converges to zero as N approaches infinity. This indicates that the rate of increase of aggregate R&D slows down and eventually levels off as the number of firms becomes very large. Finally, we show that the second derivative of aggregate R&D, $\frac{\partial^2 \mathbf{L}_z}{\partial N^2}$, is negative for large values of N. This confirms that aggregate R&D decreases after reaching a maximum point as the number of firms continues to grow.

From Equation (20), we know that

$$L_z(N,E,r) = \frac{1}{\gamma}\left[\theta \cdot \zeta(N)\frac{E}{N\xi(N)} \cdot \frac{\sigma(N)}{N-1} - \frac{r}{N-1}\right].$$

Substituting for $\xi(N) = \varepsilon - \frac{(\varepsilon-1)}{N}$, $\zeta(N) = \frac{(\varepsilon-1)(N-1)}{N}$ and $\sigma = [1+\gamma(N-1)]$ in the above equation, we have

$$L_z(N,E,r) = \frac{1}{\gamma}\left[\frac{E\theta}{N} \cdot \frac{(\varepsilon-1)(N-1)}{N\varepsilon-(\varepsilon-1)} \cdot \frac{[1+\gamma(N-1)]}{N-1} - \frac{r}{N-1}\right].$$

We obtain aggregate R&D, $\mathbf{L}_z$ by multiplying the above equation with N.

$$\mathbf{L}_z(N,E,r) = \frac{1}{\gamma}\left[E\theta \cdot \frac{(\varepsilon-1)(N-1)}{N\varepsilon-(\varepsilon-1)} \cdot \frac{[1+\gamma(N-1)]}{N-1} - \frac{rN}{N-1}\right].$$

On differentiating the aggregate R&D, $\mathbf{L}_z$, with respect to N, we obtain

$$\frac{\partial \mathbf{L}_z}{\partial N} = \frac{1}{\gamma}\left[-\theta(\varepsilon-1)\cdot E \cdot \frac{\varepsilon-\gamma}{(N\varepsilon-(\varepsilon-1))^2} + \frac{r}{(N-1)^2}\right].$$

It can be seen that $\lim_{N\to 1} \partial \mathbf{L}_z/\partial N = +\infty$ and $lim_{N\to\infty} \partial \mathbf{L}_z/\partial N = 0$.

The second order derivative of the aggregate R&D investment, $\mathbf{L}_z$, with respect to the number of firms, N, will be,

$$\frac{\partial^2 \mathbf{L}_z}{\partial N^2} = \frac{1}{\gamma}\left[\theta(\varepsilon-1)\cdot E \cdot \frac{2(\varepsilon-\gamma)\cdot\varepsilon}{(N\varepsilon-(\varepsilon-1))^3} - \frac{2r}{(N-1)^3}\right].$$

The necessary condition for the first order condition to be the maximum requires that $\frac{\partial \mathbf{L}_z}{\partial N} = 0$ and $\frac{\partial^2 \mathbf{L}_z}{\partial N^2} < 0$. The latter condition would require that

$$\frac{1}{\gamma}\left[\theta(\varepsilon-1)\cdot E\cdot\frac{2(\varepsilon-\gamma)\cdot\varepsilon}{(N\varepsilon-(\varepsilon-1))^3}-\frac{2r}{(N-1)^3}\right]<0.$$

Substituting the first order condition in the above, we obtain,

$$\frac{1}{\gamma}\left[\frac{2\varepsilon r}{(N-1)^2(N\varepsilon-(\varepsilon-1))}-\frac{2r}{(N-1)^3}\right]<0,$$

which can be re-expressed as,

$$\frac{2r}{(N-1)^2}\cdot\frac{1}{\gamma}\left[\frac{\varepsilon}{N\varepsilon-(\varepsilon-1)}-\frac{1}{N-1}\right]<0,$$

which further reduces to

$$\frac{\varepsilon}{N\varepsilon-(\varepsilon-1)}<\frac{1}{N-1}.$$

The above equation can also be expressed as,

$$1<\frac{N\varepsilon-(\varepsilon-1)}{(N-1)\varepsilon},$$

which is the same as,

$$1<\frac{\varepsilon(N-1)+1}{(N-1)\varepsilon},$$

which can be simplified to,

$$1<1+\frac{1}{(N-1)\varepsilon}.$$

With $N > 1$ and $\varepsilon > 1$, the above equation always holds, which means that $\frac{\partial^2 \mathbf{L}_z}{\partial N^2} < 0$ is always true. Therefore, for the first order condition to be the maximum, we need to have, $\frac{\partial \mathbf{L}_z}{\partial N} = 0$, which translates to

$$\frac{r}{(N-1)^2}=\theta(\varepsilon-1)\cdot E\cdot\frac{(\varepsilon-\gamma)}{(N\varepsilon-(\varepsilon-1))^2}.$$

On multiplying both sides by $(N-1)^2$, we obtain,

$$r=\theta(\varepsilon-1)\cdot E\cdot(\varepsilon-\gamma)\cdot\frac{(N-1)^2}{(N\varepsilon-(\varepsilon-1))^2},$$

which is the same as,

$$r=\theta(\varepsilon-1)\cdot E\cdot(\varepsilon-\gamma)\left[\frac{N-1}{\varepsilon(N-1)+1}\right]^2,$$

which can be rewritten as,

$$r = \theta(\varepsilon - 1) \cdot E \cdot (\varepsilon - \gamma) \left[\frac{1}{\varepsilon + \frac{1}{(N-1)})} \right]^2.$$

Note that as $N \to 1$, the right-hand side in the above equation approaches zero. And, as $N \to \infty$, the right-hand side of the above equation approaches, $\frac{\theta(\varepsilon-1)E(\varepsilon-\gamma)}{\varepsilon^2}$. Therefore, the sufficient condition for the first-order condition to be the maximum is

$$r < \frac{\theta(\varepsilon - 1)E(\varepsilon - \gamma)}{\varepsilon^2} \tag{21}$$

Intuitively, this condition implies that for aggregate R&D investment to exhibit the hump-shaped pattern, the rate of return on investment, r, must be below a certain threshold determined by parameters such as the elasticity of quality with respect to R&D investment, θ, the elasticity of substitution between varieties, ε, the total expenditure, E, and the degree of technology spillovers, γ. If r is too high, firms may not invest sufficiently in R&D, preventing the aggregate R&D investment from initially increasing and then decreasing. The threshold ensures that the environment is conducive for firms to initially increase their R&D efforts as the number of firms grows before eventually reducing them due to the dispersion of R&D resources.

If Equation (21) holds, then the aggregate R&D, $\mathbf{L}_z$, is initially increasing in the number of firms, N, and reaches its maximum at the point where $\frac{\partial \mathbf{L}_z}{\partial N} = 0$, after which it falls as the number of firms further increases. Collectively, these steps would then confirm that the aggregate R&D investment exhibits a hump-shaped pattern with respect to the number of firms, N. If, on the other hand, Equation (21) does not hold, then $\mathbf{L}_z$ is increasing in the number of firms, N. The hump shape of the aggregate R&D investment is explained as follows: as the number of firms increases significantly, R&D resources become too dispersed among the firms, preventing them from fully benefiting from economies of scale in their R&D efforts. This dispersion leads to a decrease in average R&D investment, which offsets the rise in the number of R&D projects, especially when the interest rate is sufficiently low.

2.4.3 Instantaneous Profits in the Symmetric Equilibrium

In a symmetric equilibrium, instantaneous profits are reduced to

$$\pi = (p - 1) \cdot \frac{ES}{p} - (L_z + \phi). \tag{22}$$

Using the fact that in a symmetric equilibrium, $S = 1/N$ and making use of Equation (6), the instantaneous profits are given by

$$\pi(N, E, r) = \frac{E}{N\xi(N)} - (L_z(N, E, r) + \phi)\,. \tag{23}$$

From Equation (23) we know that for a firm to make net profits, that is to say, for $\pi \geq 0$, its gross profits, $(\frac{E}{N\xi(N)})$, must be higher than the sum of its R&D investment, $L_z(N, E, r)$, and the fixed cost, ϕ. We will analyze how the instantaneous profits behave with respect to the number of firms, N, in Subsection 4.2 while discussing the producer surplus in our model.

This completes the setup of our model and the analysis of the symmetric equilibrium. In the following two sections, we proceed to define and analyze growth and welfare in the economy and study their relationship with the number of firms, N.

3 Growth

In the present study, we seek to analyze how growth and welfare are impacted by the three regimes specified earlier. This section focuses on deriving the relationship between growth and the number of firms, N. The growth rate in this economy is determined by the growth of consumption. Recall that the demand function for each variety is given by

$$x^D(p_i, q_i) = \frac{ES_i(p_i, q_i)}{p_i}. \tag{24}$$

Using the fact that in a symmetric equilibrium, $S = 1/N$ and making use of the pricing rule given Equation (6), the demand function can be rewritten as

$$x^D(p_i, q_i) = \frac{E(\xi - 1)}{N\xi}. \tag{25}$$

After plugging Equation (25) in Equation (1), we have

$$C = \left[\sum_{i=1}^{N} q_i \cdot \left(\frac{E\,(\xi - 1)}{N\xi}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{26}$$

By log linearising Equation (26), we obtain

$$log\, C(\tau) = \frac{1}{\varepsilon - 1} log\, N + log\, \frac{\xi(N) - 1}{\xi(N)} + log\, q(\tau) + log\, E(\tau). \tag{27}$$

Taking the time derivatives of Equation (27) yields[4]

$$\frac{\dot{C}}{C} = \frac{\dot{q}}{q} + \frac{\dot{E}}{E}. \tag{28}$$

Substituting for $\frac{\dot{q}}{q}$ from Equation (12), we have

$$\frac{\dot{C}}{C} = \theta[1 + \gamma(N - 1)]L_z + \frac{\dot{E}}{E}. \tag{29}$$

Multiplying and dividing the first term on the right-hand side by N, and making use of the fact that $NL_z = \mathbf{L}_z$, we get

$$\frac{\dot{C}}{C} = g(N, E, r) = \underbrace{\theta \left[\frac{1 + \gamma(N - 1)}{N}\right] \mathbf{L}_z(N, E, r)}_{(a)} + \underbrace{\frac{\dot{E}}{E}}_{(b)}. \tag{30}$$

Equation (30) gives the growth rate as a function of the number of firms in the market, N, expenditures, E, and the interest rate, r. In this economy, growth depends on how the average quality of all available brands evolves through time, which is the term (a) in Equation (30), and on the usual intertemporal trade-off between current consumption future consumption faced by consumers, which is the term (b). The evolution of quality stock over time, captured by term (a), is the same as Equation (12). The term $1 + \gamma(N - 1)$ captures

[4]Note that under the free-entry condition, firms enter the market until the point where economic profits are driven to zero. At this point, any further entry would not be profitable, and any exit would lead to positive profits for the remaining firms, encouraging entry. This results in a stable number of firms in the market. In the scenario where lobbying is considered, the policymaker sets the number of firms by granting a limited number of R&D licenses. This policy decision is based on maximizing the policymaker's utility, which is a weighted sum of consumer surplus and producer surplus. Once the number of firms is set through this political process, it remains constant as long as the policy conditions do not change. Therefore, the number of firms, N, remains constant over time, once determined.

the productivity of one unit of labor in an R&D project undertaken by the average firm and is composed of two effects: the direct effect on the quality of product developed by the firm, and the positive R&D externality. This latter effect is increasing in N since a higher number of firms allows the economy to appropriate a larger amount of spillovers. The term $\mathbf{L}_z/N$ captures the resources applied to improve the average brand of the economy. This analysis leads us to the following lemma.

Lemma 3 ***The growth rate, g, is hump-shaped in the number of firms, N, if the aggregate R&D investment, $\mathbf{L}_z/N$, is also hump-shaped. However, if $\mathbf{L}_z/N$ is monotonically increasing, the growth rate, g, may not exhibit a hump shape.***

Proof. From Equation (30), it is clear that the growth rate is influenced by the interaction of the average R&D productivity, $\frac{1+\gamma(N-1)}{N}$, the aggregate R&D investment, $\mathbf{L}_z/N$, and the intertemporal consumption trade-off, $\frac{\dot{E}}{E}$ (which is constant in the steady-state).

From Lemma 2, we have already seen that the aggregate R&D investment, $\mathbf{L}_z$ is hump-shaped. We now analyze the behavior of $\frac{1+\gamma(N-1)}{N}$ as N changes. When N is small, the denominator is small, while the numerator increases relatively slower $(N-1)$. Thus, at very small N, the average productivity, $\frac{1+\gamma(N-1)}{N}$, is high. For instance, when $N=1$,

$$\frac{1+\gamma(N-1)}{N} = \frac{1+\gamma(1-1)}{1} = 1.$$

As N increases, the numerator, $[1+\gamma(N-1)]$, grows linearly, but the denominator grows faster. Consequently, $\frac{1+\gamma(N-1)}{N}$ decreases. Thus, the function $\frac{1+\gamma(N-1)}{N}$ decreases monotonically with N. It starts at a high value of 1 when $N=1$ and asymptotically approaches $0<\gamma<1$ as $N\to\infty$. This reflects the diminishing returns to R&D productivity per firm as the number of firms grows. Thus, when $N\to\infty$,

$$\frac{1+\gamma(N-1)}{N} \to \gamma.$$

To formally determine whether g is hump-shaped, we differentiate it with respect to N. We obtain

$$\frac{\partial g}{\partial N} = \theta\left[-\frac{1-\gamma}{N^2}\mathbf{L}_z + \frac{1+\gamma(N-1)}{N}\frac{\partial \mathbf{L}_z}{\partial N}\right].$$

From Lemma 2, it is known that $\mathbf{L}_z$ is hump-shaped. This means that $\frac{\partial \mathbf{L}_z}{\partial N}$ is positive for small values of N and negative for larger values of N.

For small N, the first term in the above equation is small in magnitude, while the second term is positive, making $\frac{\partial g}{\partial N} > 0$, meaning g is increasing. For large N, the first term remains negative, while the second term becomes negative (since $\frac{\partial \mathbf{L}_z}{\partial N}$ turns negative), making $\frac{\partial g}{\partial N} < 0$, meaning g is decreasing. Therefore, g is hump-shaped.

However, if $\mathbf{L}_z$ is monotonically increasing, the sign of $\frac{\partial g}{\partial N}$ depends on the magnitude of $\frac{\partial \mathbf{L}_z}{\partial N}$. If $\frac{\partial \mathbf{L}_z}{\partial N}$ is large enough, it may offset the decreasing effect of $\frac{1+\gamma(N-1)}{N}$, preventing g from exhibiting a hump shape. Thus, g is hump-shaped if $\mathbf{L}_z$ is hump-shaped, but it may not be if $\mathbf{L}_z$ is strictly increasing.

We next proceed to define welfare in this economy and analyze its relationship with the number of firms, N.

4 Welfare

Another aspect of interest in our present study is the welfare outcomes in the three specified regimes. Júlio (2014) defines welfare as the lifetime utility of the representative consumer, $U(N, E, r)$[5], which is influenced by three main effects. (1) The love for variety effect, $\frac{1}{\rho} \cdot \frac{1}{\varepsilon-1} \, log \, N$. This captures the fact that the consumers derive utility from a greater number

[5]In Júlio (2014), the lifetime utility of the representative household is expressed as

$$U(N, E, r) = \frac{1}{\rho}\left[\frac{1}{\varepsilon - 1} log \, N + log \, \frac{\xi(N) - 1}{\xi} + \frac{g(N, E, r)}{\rho} + log \, E\right].$$

This equation demonstrates that welfare increases with N due to the variety and competition effects, but this relationship may be offset by the growth effect, which can be hump-shaped with respect to N. To ensure welfare increases monotonically with N, Júlio introduces an assumption which requires

$$\frac{1}{\rho}\left[\frac{1}{\varepsilon - 1} \, log \, N + log \, \frac{\xi(N) - 1}{\xi} + \frac{g(N, E, r)}{\rho} + log \, E\right] > 0.$$

This assumption ensures that the growth effect The assumption plays a pivotal role in Júlio's analysis by ensuring that welfare is always increasing in N. Without it, the growth effect could outweigh the variety and competition effects, leading to a potential non-monotonic relationship between welfare and the number of firms. This condition guarantees that the *laissez-faire* equilibrium is welfare-maximizing, given E.

of varieties. (2) The competition effect, $log\,\frac{\xi(N)-1}{\xi(N)}$. This reflects the fact that increased competition due to a larger number of firms reduces individual firms' markups. (3) The growth effect, $\frac{g(N,E,r)}{\rho}$. This accounts for the increase in the aggregate utility of welfare over time due to economic growth.

These effects jointly determine welfare outcomes in Júlio's model. However, Júlio's analysis does not distinguish between consumer and producer welfare, potentially overlooking important trade-offs through which the key results are transmitted. Our framework extends the welfare analysis by explicitly accounting for both consumer and producer surplus. Aggregate consumer surplus, CS, measures the net benefits derived by consumers after accounting for their expenditures, while total producer surplus, Π, captures the net profits of firms after deducting production and R&D costs. This allows us to assess how different regimes or market structures affect the distribution of welfare between consumers and producers. Accordingly, we define welfare as

$$W = CS + \Pi. \tag{31}$$

By adopting this broader definition, our model provides a more comprehensive basis for evaluating policy interventions, market outcomes, and the trade-offs through which these outcomes result. In particular, we analyze how changes in the number of firms, N, affect aggregate welfare and its components under the three regimes specified earlier in this study.

We now proceed to define and analyze each of the two components of welfare in our economy.

4.1 Consumer Surplus

The first component of welfare is the aggregate or economy-wide consumer surplus, CS, which reflects the difference between consumers' willingness to pay for goods and the actual prices they pay. This is given by

$$CS = N \cdot \int_{p_i}^{\infty} x_i \, dp_i, \tag{32}$$

where x_i denotes the demand for a typical variety i, and p_i is the price of that variety. From the utility maximization exercise, we know

$$x_i = E \cdot \frac{p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}}{p_i \cdot \sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}}. \tag{33}$$

Substituting p_i from Equation (6) and x_i from Equation (33) in Equation (32) we have

$$CS = N \cdot \int_{p_i}^{\infty} E \cdot \frac{p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}}{p_i \cdot \sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}} \, dp_i. \tag{34}$$

Equation (34) calculates the total benefit consumers get from purchasing all varieties of goods in the market. The term within the integral sign represents the quantity demanded for a variety, i, taking into account the price, p_i, and quality, q_i, of that variety relative to other varieties in the market. This term shows that demand depends on both the individual product's price and quality, as well as the prices and qualities of all other products. The integral computes the consumer surplus for a single variety, i, by summing up the surpluses for different possible prices. It reflects how much more consumers value the good compared to its market price, aggregated across all possible prices above p_i. The multiplication by N scales this individual surplus by the number of varieties in the market, thus providing the total consumer surplus summed up across all product varieties. This multiplication is due to the symmetry in the economy, which ensures that the consumer surplus for each variety is identical, allowing for a straightforward scaling to obtain the aggregate surplus.

Evaluating the integral in Equation (34) yields

$$CS = NE \cdot \frac{q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}} \cdot \frac{1}{\varepsilon - 1} \left[\frac{\xi(N) - 1}{\xi(N)} \right]^{(\varepsilon-1)}. \tag{35}$$

Next, substituting for $\xi = \varepsilon - \frac{\varepsilon-1}{N}$, which is the case in a symmetric equilibrium, we get

$$CS = NE \cdot \frac{q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}} \cdot \frac{1}{\varepsilon - 1} \left[\frac{\varepsilon - \frac{\varepsilon-1}{N} - 1}{\varepsilon - \frac{\varepsilon-1}{N}} \right]^{(\varepsilon-1)}, \tag{36}$$

which simplifies to

$$CS = NE \cdot \frac{q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}} \cdot \frac{1}{\varepsilon - 1} \left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon - (\varepsilon - 1)} \right]^{(\varepsilon-1)}. \tag{37}$$

Note that the term $\frac{q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}}$ is the quality of i^{th} firm, relative to the average of qualities of each of the N firms operating in the economy, weighted by the respective prices.

Consumers perceive this relative quality while exhibiting their preference for the variety of goods available in the market.

We now analyze how the consumer surplus behaves with respect to N. We have,

$$\frac{\partial CS}{\partial N} = \frac{E \cdot q_i^{(\varepsilon-1)}}{(\varepsilon-1)\cdot\sum_{j=1}^{N} p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}} \Bigg\{ \left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{(\varepsilon-1)} \cdot 1 +$$

$$N\cdot(\varepsilon-1)\left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{\varepsilon-2} \cdot \left[\frac{(N\varepsilon-(\varepsilon-1))(\varepsilon-1)-(N-1)(\varepsilon-1)\cdot\varepsilon}{[N\varepsilon-(\varepsilon-1)]^2}\right]\Bigg\}. \quad (38)$$

This can be re-expressed to yield

$$\frac{\partial CS}{\partial N} = \frac{E \cdot q_i^{(\varepsilon-1)}}{(\varepsilon-1)\cdot\sum_{j=1}^{N} p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}} \cdot \left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{(\varepsilon-1)}$$

$$\Bigg\{1 \ + N\cdot(\varepsilon-1)\left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{-1} \cdot (\varepsilon-1) \cdot \left[\frac{(N\varepsilon-(\varepsilon-1))-(N-1)\cdot\varepsilon}{[N\varepsilon-(\varepsilon-1)]^2}\right]\Bigg\}, \quad (39)$$

which can further be simplified to

$$\frac{\partial CS}{\partial N} = \frac{E \cdot q_i^{(\varepsilon-1)}}{(\varepsilon-1)\cdot\sum_{j=1}^{N} p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}} \cdot \left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{(\varepsilon-1)}$$

$$\Bigg\{1 \ + N\cdot(\varepsilon-1)\left[\frac{N\varepsilon-(\varepsilon-1)}{(N-1)(\varepsilon-1)}\right] \cdot (\varepsilon-1) \cdot \left[\frac{N\varepsilon-\varepsilon+1-N\varepsilon+\varepsilon}{[N\varepsilon-(\varepsilon-1)]^2}\right]\Bigg\}, \quad (40)$$

which on canceling out terms within the parentheses would reduce to

$$\frac{\partial CS}{\partial N} = \frac{E \cdot q_i^{(\varepsilon-1)}}{(\varepsilon-1)\cdot\sum_{j=1}^{N} p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}} \cdot \left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{(\varepsilon-1)}$$

$$\Bigg\{1 \ + N\cdot(\varepsilon-1)\left[\frac{1}{(N-1)}\right] \cdot \left[\frac{1}{[N\varepsilon-(\varepsilon-1)]}\right]\Bigg\}. \quad (41)$$

By rearrangement of terms, it can be written as

$$\frac{\partial CS}{\partial N} = \frac{E \cdot q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N} p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}} \cdot \left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{(\varepsilon-1)} \cdot \left[\frac{1}{(\varepsilon-1)} + \frac{N}{(N-1)(N\varepsilon-(\varepsilon-1))}\right]. \quad (42)$$

Upon multiplying and dividing the right-hand side by $p_i^{-(\varepsilon-1)}$, we obtain

$$\frac{\partial CS}{\partial N} = \frac{E \cdot q_i^{(\varepsilon-1)}}{N \cdot p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}} \cdot \frac{p_i^{-(\varepsilon-1)}}{p_i^{-(\varepsilon-1)}} \cdot \left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{(\varepsilon-1)} \cdot \left[\frac{1}{(\varepsilon-1)} + \frac{N}{(N-1)(N\varepsilon-(\varepsilon-1))}\right]. \quad (43)$$

We substitute for $S(p_i, q_i) = \frac{p_i^{-(\varepsilon-1)} q_i^{\varepsilon-1}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{\varepsilon-1}}$, in the above equation, to obtain

$$\frac{\partial CS}{\partial N} = \frac{ES}{p^{-(\varepsilon-1)}} \cdot \left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{(\varepsilon-1)} \left[\frac{1}{(\varepsilon-1)} + \frac{N}{(N-1)(N\varepsilon-(\varepsilon-1))}\right]. \quad (44)$$

Invoking symmetric equilibrium, we drop the subscript over the price, and also substitute for $S(p_i, q_i)$, to obtain

$$\frac{\partial CS}{\partial N} = \frac{E}{N \cdot p^{-(\varepsilon-1)}} \cdot \left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{(\varepsilon-1)} \left[\frac{1}{(\varepsilon-1)} + \frac{N}{(N-1)(N\varepsilon-(\varepsilon-1))}\right]. \quad (45)$$

We know that $p = \frac{\xi}{\xi-1} = \frac{N\varepsilon-(\varepsilon-1)}{(N-1)(\varepsilon-1)}$ and therefore

$$p^{-(\varepsilon-1)} = \left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{(\varepsilon-1)}.$$

Thus, Equation (45) can now be written as

$$\frac{\partial CS}{\partial N} = \frac{E}{N \cdot \left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{(\varepsilon-1)}} \cdot \left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{(\varepsilon-1)} \cdot \left[\frac{1}{(\varepsilon-1)} + \frac{N}{(N-1)(N\varepsilon-(\varepsilon-1))}\right], \quad (46)$$

which simplifies to

$$\frac{\partial CS}{\partial N} = \frac{E}{N}\left[\frac{1}{(\varepsilon-1)} + \frac{N}{(N-1)(N\varepsilon-(\varepsilon-1))}\right]. \quad (47)$$

Equation (47) can also be written as

$$\frac{\partial CS}{\partial N} = E\Bigg[\underbrace{\frac{1}{N(\varepsilon-1)}}_{(a)} + \underbrace{\frac{1}{(N-1)(N\varepsilon-(\varepsilon-1)}}_{(b)}\Bigg] > 0. \tag{48}$$

Since $\varepsilon > 1$, $N > 1$ and $E > 0$, it can be seen from Equation (48) that the overall effect of an increase in N on consumer surplus is unambiguously positive, that is to say, $\frac{\partial CS}{\partial N} > 0$. Note that there are two effects through which a change in market concentration affects consumer surplus. $Term\ (a)$ captures the love for variety effect. Since consumers exhibit a love for variety, a decrease in market concentration, that is to say, an increase in N, makes consumers better off, since there is a larger number of varieties available to the consumer. $Term\ (b)$ captures the competition effect. This flows from the fact that when there is a decrease in market concentration, the markup price also decreases, thus again benefiting the consumers by making goods cheaper. Equation (48) leads us to note the following lemma.

Lemma 4 ***The consumer surplus increases with an increase in the number of firms, N. However, the rate of increase diminishes as N grows, making consumer surplus a concave function of N.***

The first part of the lemma follows directly from Equation (48) and the discussion thereupon, which shows that $\frac{\partial CS}{\partial N}$ is positive. Further, to check the rate at which consumer surplus increases with an increase in the number of varieties, N, we differentiate Equation (48) with respect to N.

$$\frac{\partial^2 CS}{\partial N^2} = E\Bigg[-\frac{1}{(\varepsilon-1)N^2} - \frac{(N\varepsilon-(\varepsilon-1))\cdot 1 + (N-1)\cdot\varepsilon}{[(N-1)(N\varepsilon-(\varepsilon-1))]^2}\Bigg], \tag{49}$$

which can be simplified to

$$\frac{\partial^2 CS}{\partial N^2} = E\Bigg[-\frac{1}{(\varepsilon-1)N^2} - \frac{\varepsilon(N-1)+1+\varepsilon(N-1)}{[(N-1)(N\varepsilon-(\varepsilon-1))]^2}\Bigg], \tag{50}$$

which can further be simplified to

$$\frac{\partial^2 CS}{\partial N^2} = E\Bigg[-\frac{1}{(\varepsilon-1)N^2} - \frac{2\varepsilon(N-1)+1}{[(N-1)(N\varepsilon-(\varepsilon-1))]^2}\Bigg] < 0. \tag{51}$$

Again, since $\varepsilon > 1$, $N > 1$ and $E > 0$, it can be seen from Equation (51) that $\frac{\partial^2 CS}{\partial N^2} < 0$. Therefore, consumer surplus increases at a decreasing rate with a rise in the number of varieties, N. In other words, consumer surplus is a concave function of N. The concavity of CS with respect to N implies diminishing marginal benefits to consumer welfare from increasing the number of firms. Beyond a certain point, additional firms contribute less to consumer surplus due to diminishing returns from both the love for variety and competition effects.

Having defined the aggregate consumer surplus in the economy and analyzing its relationship with the number of firms, N, we now proceed to define aggregate producer surplus and analyze its relationship with the number of firms, N.

4.2 Producer Surplus

The other component of welfare is the aggregate producer surplus. In this section, we analyze the aggregate producer surplus, which is defined as the sum of profits made by all the firms put together. Hence,

$$\Pi = N \cdot \pi. \tag{52}$$

Substituting for π from Equation (23), we have

$$\Pi = \frac{E}{\xi(N)} - NL_z - N\phi. \tag{53}$$

Next, substituting for L_z from Equation (20)

$$\Pi = \frac{E}{\xi(N)} - N\frac{1}{\gamma}\left[\theta \cdot \zeta(N)\frac{E}{N\xi(N)} \cdot \frac{\sigma(N)}{N-1} - \frac{r}{N-1}\right] - N\phi. \tag{54}$$

Further, by substituting for $\sigma(N) = [1 + \gamma(N-1)]$ from Equation (12), the price elasticity, $\xi(N) = \varepsilon - \frac{(\varepsilon-1)}{N}$, and quality elasticity, $\zeta(N) = \frac{(\varepsilon-1)\cdot(N-1)}{N}$, in Equation (54), and by rearranging and canceling out terms, we obtain

$$\Pi = \frac{N \cdot E}{N\varepsilon - (\varepsilon - 1)} - E\theta \cdot \frac{(\varepsilon-1)[1+\gamma(N-1)]}{\gamma[N\varepsilon - (\varepsilon-1)]} + \frac{N \cdot r}{\gamma(N-1)} - N\phi. \tag{55}$$

We next consider how the producer surplus changes with change in N. We get that

$$\frac{\partial \Pi}{\partial N} = E \cdot \left[\frac{(N\varepsilon - (\varepsilon - 1)) \cdot 1 - N \cdot \varepsilon}{(N\varepsilon - (\varepsilon - 1))^2}\right]$$

$$- \frac{E\theta(\varepsilon - 1)}{\gamma} \cdot \left[\frac{(N\varepsilon - (\varepsilon - 1)) \cdot \gamma - (1 + \gamma(N-1)) \cdot \varepsilon}{(N\varepsilon - (\varepsilon - 1))^2}\right]$$

$$+ \frac{r}{\gamma} \cdot \left[\frac{(N-1) \cdot 1 - N \cdot 1}{(N-1)^2}\right] - \phi. \quad (56)$$

This can be simplified to

$$\frac{\partial \Pi}{\partial N} = E \cdot \left[\frac{-(\varepsilon - 1))}{(N\varepsilon - (\varepsilon - 1))^2}\right] - \frac{E\theta(\varepsilon - 1)}{\gamma} \cdot \left[\frac{N\varepsilon\gamma - \varepsilon\gamma + \gamma - \varepsilon - N\varepsilon\gamma + \varepsilon\gamma}{(N\varepsilon - (\varepsilon - 1))^2}\right]$$

$$+ \frac{r}{\gamma} \cdot \left[\frac{N - 1 + N}{(N-1)^2}\right] - \phi, \quad (57)$$

which further simplifies to

$$\frac{\partial \Pi}{\partial N} = -E \cdot \left[\frac{(\varepsilon - 1))}{(N\varepsilon - (\varepsilon - 1))^2}\right] - \frac{E\theta(\varepsilon - 1)}{\gamma} \cdot \left[\frac{\gamma - \varepsilon}{(N\varepsilon - (\varepsilon - 1))^2}\right] - \frac{r}{\gamma(N-1)^2} - \phi. \quad (58)$$

On collecting like terms on the right-hand side, the above equation yields

$$\frac{\partial \Pi}{\partial N} = -\frac{E(\varepsilon - 1)}{(N\varepsilon - (\varepsilon - 1))^2}\left[1 + \frac{\theta}{\gamma} \cdot (\gamma - \varepsilon)\right] - \frac{r}{\gamma(N-1)^2} - \phi, \quad (59)$$

which can also be written as

$$\frac{\partial \Pi}{\partial N} = \underbrace{\underbrace{-\frac{E(\varepsilon - 1)}{(N\varepsilon - (\varepsilon - 1))^2}\left[1 - \frac{\theta}{\gamma} \cdot (\varepsilon - \gamma)\right]}_{(c)} \underbrace{-\frac{r}{\gamma(N-1)^2}}_{(d)}}_{(a)} \underbrace{-\phi}_{(b)}.$$

The change in aggregate producer surplus with respect to N can be decomposed into two components. The first term, (a), is the change in contribution margin due to a change in N. We define contribution margin as the difference between sales revenue and variable cost[6]. The second term, (b), is the change in average fixed cost due to a change in N.

[6]The concept of contribution margin is key to break-even analysis. It represents the portion of sales revenue that exceeds variable costs, thereby contributing to the coverage of fixed costs and profit generation. It is calculated as sales revenue minus variable costs and is often expressed as a percentage of sales revenue, known as the contribution margin ratio [Farris et al. (2010)]. This metric is instrumental in assessing profitability, setting pricing strategies, and making cost management decisions.

Term (b) is negative since $\phi > 0$, implying that higher fixed costs reduce aggregate producer surplus. With $r > 0$, $\gamma \in (0, 1)$ and $N > 1$, it is evident that term (d) is also negative. However, the sign of (c) depends on the behavior of θ. While parameters such as $E > 0$, $\varepsilon > 1$, $N > 1$ and $\gamma \in (0, 1)$ are known, the sign of $\left(1 - \frac{\theta(\varepsilon-\gamma)}{\gamma}\right)$ is indeterminate without additional assumptions regarding θ. If term (c) dominates the negative contributions of (d) and (b), $\frac{\partial \Pi}{\partial N}$ could potentially become positive, leading to the counterintuitive outcome of increasing aggregate profits with higher N. This perverse effect of N on aggregate profits can be eliminated by introducing the following assumption.

Assumption 1 ***The elasticity of product quality with respect to R&D investments, θ, is sufficiently small relative to the degree of spillovers in the economy, γ, and the elasticity of substitution between products, ε. Formally, this condition is expressed as***

$$\theta < \frac{\gamma}{\varepsilon - \gamma}.$$

This assumption ensures that R&D productivity, θ, does not disproportionately incentivize innovation, which could otherwise lead to unintended equilibrium outcomes, such as increasing aggregate profits with higher N. Specifically, this assumption guarantees that the term $\left(1 - \frac{\theta(\varepsilon-\gamma)}{\gamma}\right)$ remains positive. This, in turn, ensures that term (c) in the above equation is rendered negative, and, consequently, $\frac{\partial \Pi}{\partial N}$ is negative. This implies that aggregate producer surplus decreases monotonically with N, thereby simplifying the subsequent analysis. By ensuring that aggregate profits are decreasing in N, we also ensure equilibrium uniqueness in the free entry and exit regime where the number of firms, N, is determined through the free entry and exit of firms and the zero-profit condition. Therefore, by virtue of Assumption 1, the derivative of aggregate producer surplus with respect to N is given by

$$\frac{\partial \Pi}{\partial N} = -\frac{E(\varepsilon - 1)}{(N\varepsilon - (\varepsilon - 1))^2}\left[1 - \frac{\theta}{\gamma} \cdot (\varepsilon - \gamma)\right] - \frac{r}{\gamma(N-1)^2} - \phi < 0 \tag{60}$$

This result indicates that aggregate producer surplus decreases with an increase in the number of firms N.

Next, we examine the rate at which producer surplus decreases as N changes by computing the second derivative of Π with respect to N. Thus, differentiating Equation (60) with

Component	First-order Condition	Second-order Condition
Consumer Surplus	$\frac{\partial CS}{\partial N} > 0$ *[Equation (48)]*	$\frac{\partial^2 CS}{\partial N^2} < 0$ *[Equation (51)]*
Producer Surplus	$\frac{\partial \Pi}{\partial N} < 0$ *[Equation (60)]*	$\frac{\partial^2 \Pi}{\partial N^2} > 0$ *[Equation (61)]*

Table 1: Summary of Relationships of Consumer Surplus and Producer Surplus with the Number of Firms, N.

respect to N yields

$$\frac{\partial^2 \Pi}{\partial N^2} = \underbrace{\underbrace{\frac{2E(\varepsilon-1)\varepsilon}{(N\varepsilon-(\varepsilon-1))^3}}_{(c)} \underbrace{\left[1 - \frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]}_{(d)}}_{(a)} + \underbrace{\frac{2r}{\gamma(N-1)^3}}_{(b)} > 0. \tag{61}$$

Given $E > 0$, $\varepsilon > 1$, $N > 1$, $\gamma \in (0,1)$, terms (b) and (c) are positive. And by virtue of Assumption 1, term (d) is also positive, making the overall term (a) positive. Consequently, the second derivative $\frac{\partial^2 \Pi}{\partial N^2}$ is strictly positive. This result implies that as the number of firms N increases, aggregate producer surplus decreases at an increasing rate. The convexity of the producer surplus curve with respect to N reflects the compounding effects of increasing fixed costs, competitive pressures, and diminishing returns to R&D.

We summarize the relationship of consumer surplus and producer surplus with the number of firms, N, in Table 1. Having established the behavior of aggregate consumer surplus and producer surplus, we now turn our attention to defining overall welfare in the model and analyzing its relationship with N.

4.3 Aggregate Welfare

As defined earlier, aggregate welfare, W, in the economy is the combination of total consumer surplus, CS, and total producer surplus Π. Using the expressions derived in Equation (37) and Equation (55), the aggregate welfare of the economy can be expressed as

$$W(N,E,r) = NE \cdot \frac{q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}} \cdot \frac{1}{\varepsilon-1}\left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{(\varepsilon-1)}$$

$$+ \frac{N\cdot E}{N\varepsilon-(\varepsilon-1)} - E\theta\cdot\frac{(\varepsilon-1)[1+\gamma(N-1)]}{\gamma[N\varepsilon-(\varepsilon-1)]} + \frac{N\cdot r}{\gamma(N-1)} - N\phi. \tag{62}$$

Equation (62) represents welfare as a function of three key variables: the number of firms, N, expenditures, E and the interest rate, r. Each term reflects the interplay of consumer and producer behavior in determining overall welfare. The first term captures the benefits consumers derive from variety and competition. The subsequent terms account for the revenue generated by firms, their R&D and production costs, and fixed costs. Together, they provide a comprehensive measure of aggregate welfare in the economy.

To understand how changes in the number of firms, N, affect aggregate welfare, we compute the first derivative of W as

$$\frac{\partial W}{\partial N} = \frac{\partial CS}{\partial N} + \frac{\partial \Pi}{\partial N}. \tag{63}$$

From Equation (48) and Equation (60), we know the signs of the changes in consumer surplus and producer surplus due to a change in N are

$$\frac{\partial W}{\partial N} = \underbrace{\frac{\partial CS}{\partial N}}_{>0} + \underbrace{\frac{\partial \Pi}{\partial N}}_{<0} \gtrless 0. \tag{64}$$

Thus, the overall effect that a change in the number of varieties or firms in the economy, N, has on the aggregate welfare, W, cannot be ascertained unambiguously. Critically, it depends on whether the gain in consumer surplus outweighs the loss in producer surplus. Once the relationship between welfare and the number of firms is established, the value of W under each regime can be determined by solving for the optimal N in each case. This allows for a meaningful comparison of welfare outcomes across the three regimes.

By substituting Equation (48) and Equation (60) in Equation (64) we have,

$$\frac{\partial W}{\partial N} = \underbrace{\frac{E}{N}\left[\frac{1}{(\varepsilon-1)} + \frac{N}{(N-1)(N\varepsilon-(\varepsilon-1))}\right]}_{(a)} - \underbrace{\left\{\left[\frac{E(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right] + \frac{r}{\gamma(N-1)^2}\right] + \phi\right\}}_{(b)} \gtrless 0. \tag{65}$$

Observe that for $\frac{\partial W}{\partial N} > 0$, we need to have that the positive term, (a), should outweigh the negative term, (b). This highlights the trade-offs inherent in determining the welfare-maximizing N: while increasing the number of firms benefits consumers through greater variety and lower prices, it imposes costs on producers through reduced profitability.

The second derivative of welfare provides insights into the rate at which welfare, W, changes with the number of firms, N. This is given by

$$\frac{\partial^2 W}{\partial N^2} = \underbrace{\frac{\partial^2 CS}{\partial N^2}}_{<0} + \underbrace{\frac{\partial^2 \Pi}{\partial N^2}}_{>0} \gtrless 0. \tag{66}$$

From Equation (51), we know that consumer surplus increases at a decreasing rate with N, reflecting diminishing marginal gains from variety and competition. Similarly, based on Assumption 1, Equation (61) states that the producer surplus decreases at an increasing rate with N, due to rising fixed costs and diminishing returns to scale. Thus, the rate of change of W, due to a change in N, is again indeterminate.

In our model, one of the firm-entry regimes is the purely benevolent policymaker regime, where the firm-entry policy, N, is set by maximizing overall welfare. For this policymaker to determine the welfare-maximizing N, the welfare function, W, must exhibit concavity with respect to N. Given these theoretical and mathematical considerations, we make the following assumption.

Assumption 2 ***The aggregate welfare, W, is concave in the number of firms, N. Formally,***

$$\frac{\partial^2 W}{\partial N^2} = \underbrace{\frac{\partial^2 CS}{\partial N^2}}_{<0} + \underbrace{\frac{\partial^2 \Pi}{\partial N^2}}_{>0} < 0, \tag{67}$$

This assumption ensures that W is concave in N, allowing for a unique welfare-maximizing level of market concentration. It reflects the expectation that excessive entry reduces overall welfare by imposing disproportionate costs on producers while providing diminishing benefits to consumers. This assumption is both analytically convenient and consistent with the theoretical expectation that welfare achieves a unique maximum at some intermediate value of N[7].

[7] The assumption of concavity in the welfare function with respect to the number of varieties is fundamentally rooted in the Dixit and Stiglitz (1977) framework. This model employs a CES (constant elasticity of substitution) utility function to formalize consumer preferences, where consumers derive utility from product variety. The diminishing marginal utility from additional varieties in this model reflects a saturation effect: as the market becomes increasingly diverse, the incremental welfare gains from introducing a new

By establishing the concavity of W and its dependence on N, we create a foundation for evaluating the welfare-maximizing N under different regimes. In the following sections, we analyze the implications of this framework for the three regimes considered in our model, namely, the free entry and exit regime, the benevolent policymaker regime, and the politically motivated policymaker regime.

5 The Benchmark: Equilibrium with no Lobbying

We first characterize the equilibrium for an unregulated market economy, or the *laissez-faire* economy. It would be pertinent to note here the process in which general equilibrium is achieved in this economy. First, the industry equilibrium is attained through the free entry and exit condition, which ensures that instantaneous profits are zero at all times.

As the market structure evolves, individual decisions made by firms also adjust, primarily due to the externality induced by positive spillovers of R&D efforts among firms. These adjustments in R&D decisions lead to imbalances in the labor market, creating a disequilibrium. This disequilibrium necessitates corrections through changes in labor market allocations. General equilibrium is ultimately achieved when the labor market clears, ensuring that total labor demand equals total labor supply.

We begin with characterizing the industry equilibrium under the free entry and exit regime.

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variety diminish. This concavity ensures that there exists an optimal, finite level of diversity that maximizes aggregate welfare, balancing consumers' desire for variety against the costs associated with producing and sustaining differentiated goods.

Aghion and Howitt (1990) extend this principle into the dynamic setting of endogenous growth theory. In their model of growth through creative destruction, innovation acts as the engine of economic expansion by introducing new products or technologies. However, they demonstrate that the marginal benefits of innovation diminish as the economy accumulates technological improvements, reflecting a concave relationship between innovation and welfare in the long run. This concavity also highlights the inherent trade-off in allocating resources between the development of new products and the optimization of existing production processes. By integrating these concepts, the two frameworks offer complementary perspectives on the role of variety in shaping static and dynamic welfare outcomes.

5.1 Industry Equilibrium

With free entry and no intervention by the policymaker, the number of firms, N, adjusts instantaneously to ensure equilibrium conditions are satisfied. Specifically, free entry and exit condition ensures that whenever $V > 0$, new firms enter the market, and whenever $V < 0$, firms exit the market. This dynamic ensures that the system remains in equilibrium at all times.

Differentiating Equation (4) with respect to time and rearranging the terms, we obtain the following perfect foresight no-arbitrage condition for the equilibrium in the capital market

$$rV = \pi + \dot{V}.$$

In the context of free entry, the entry condition requires $V = 0$. Substituting $V = 0$ into the above equation implies that instantaneous profits, $\pi(N, E, r)$, as characterized by Equation (23), must equal zero at all times. Therefore, profits are determined as

$$\pi(N, E, r) = \frac{E}{N\xi(N)} - (L_z(N, E, r) + \phi) = 0, \tag{68}$$

which reduces to

$$\frac{E}{N\xi(N)} = L_z(N, E, r) + \phi. \tag{69}$$

Equation (32) determines the equilibrium number of firms as a function of aggregate expenditures, E, and the interest rate, r, with L_z given by Equation (20). Let $N^f(E, r)$ denote the solution to Equation (32), representing the equilibrium number of firms in the free entry and exit regime. Equation (32) can then be rewritten in terms of aggregate R&D as

$$N^f(E, r) \cdot L_z = \mathbf{L}_z = \frac{E}{\xi(N^f(E, r))} - N^f(E, r) \cdot \phi. \tag{70}$$

Having characterized the industry equilibrium under the free entry and exit regime, we now turn our attention to the general equilibrium, where labor market clearing and aggregate resource constraints must also be satisfied.

5.2 General Equilibrium

To achieve general equilibrium, the labor market clearing condition and aggregate resource constraints must be satisfied alongside the industry equilibrium conditions derived earlier.

Let E^f denote the equilibrium level of expenditures. The labor market clearing condition is given by

$$N^f\left(E^f,r\right)\cdot L_X\left(N^f\left(E^f,r\right),E^f,r\right)+\quad \mathbf{L}_z\left(N^f\left(E^f,r\right),E^f,r\right)=1, \tag{71}$$

where $\mathbf{L}_z$ captures the industry-level labor demand for R&D, and is given by Equation (70). L_x represents the firm-level labor demand for production and is given by Equation (2) as

$$L_x = X+\phi. \tag{72}$$

In equilibrium, the total output produced by a firm, X, must equal the aggregate demand for its variety, which originates from all representative households in the economy. Since we assume symmetry across all firms and households, the demand for a single variety by a representative household, $x^D\left(p_i,q_i\right)$ equals the firm's total output. Thus, substituting for X from Equation (25) into Equation (72), we obtain the labor demand for production as

$$L_x(N^f(E^f,r),E^f) = E^f\cdot\frac{\xi(N^f(E^f,r))-1}{N^f(E^f,r)\cdot\xi(N^f(E^f,r))}+\phi. \tag{73}$$

Substituting Equation (70) and Equation (73) in Equation (71), we obtain

$$N^f(E^f,r)\cdot\left[E^f\cdot\frac{\xi(N^f(E^f,r))-1}{N^f(E^f,r)\cdot\xi(N^f(E^f,r))}+\phi\right] \\ +\frac{E^f}{\xi(N^f(E^f,r))}-N^f(E^f,r)\cdot\phi=1. \tag{74}$$

This simplifies to

$$E^f=1. \tag{75}$$

This result implies that equilibrium expenditures are normalized to unity in the general equilibrium. Therefore, in the general equilibrium

$$\frac{\dot{E}}{E}=r-\rho=0, \tag{76}$$

which then implies that $r=\rho$. This ensures intertemporal consistency in household decisions and a flat path of expenditure along the steady state.

By substituting Equation (69) and Equation (76) and the quality elasticity of demand, $\zeta=\frac{(\varepsilon-1)\cdot(N-1)}{N}$, in Equation (20) we have

$$L_z=\frac{1}{\gamma}\left[\theta\left(\frac{(\varepsilon-1)\cdot(N^f-1)}{N^f}\right)\cdot\frac{1}{N^f\left(\varepsilon-\frac{(\varepsilon-1)}{N^f}\right)}\cdot\frac{\sigma(N^f)}{N^f-1}-\frac{\rho}{N^f-1}\right], \tag{77}$$

which can be further reduced to

$$\frac{1}{N^{fe} \cdot \xi(N^{fe})} \left[1 - \frac{\theta(\varepsilon - 1)\sigma(N^{fe})}{\gamma \cdot N^{fe}}\right] + \frac{\rho}{\gamma \cdot (N^{fe} - 1)} = \phi.$$

Recall that $\xi(N) = \varepsilon - \frac{(\varepsilon-1)}{N}$ and $\sigma = [1 + \gamma(N - 1)]$. Thus, the above equation can now be written as

$$\frac{1}{N^{fe}\varepsilon - (\varepsilon - 1)} \left[1 - \frac{\theta}{\gamma} \cdot \frac{(\varepsilon - 1)[1 + \gamma(N^{fe} - 1)])}{N^{fe}}\right] + \frac{\rho}{\gamma \cdot (N^{fe} - 1)} = \phi. \tag{78}$$

The equilibrium number of firms under free entry, N^{fe}, is the solution to Equation (78). Note that $\lim_{N\to 1}$ of the left-hand side in the above equation is ∞ and $\lim_{N\to\infty}$ of the left-hand side in the above equation is 0.

[Refer to Appendix 4 for the proof.]

Note that higher fixed costs, ϕ, reduce N^{fe}, as the equilibrium requires larger revenues per firm to offset fixed costs. Higher spillovers, γ, increase N^{fe} by reducing the effective cost of R&D, allowing more firms to operate profitably. Higher elasticity of substitution, ε, increases N^{fe}, by intensifying competition and reducing markups. The equilibrium ties together labor allocation, expenditures, and firm entry, ensuring that all markets clear, and resources are fully utilized.

This concludes the characterization of the general equilibrium under the free entry and exit regime. Next, we proceed to analyze the implications of this equilibrium on economic growth and welfare within the economy.

5.3 Equilibrium Growth and Welfare

Equilibrium growth and welfare under the free entry and exit regime are derived by substituting the equilibrium values of N, E and r into the growth and welfare functions.

Equilibrium growth, g^f, is obtained by replacing the equilibrium values of $N = N^{fe}$, and $E = E^f = 1$, in the growth Equation (30), and letting $\mathbf{L}_z^f = \mathbf{L}_z(N^{fe}, E^f)$. This yields

$$g^f = \theta \left[\frac{1 + \gamma(N^{fe} - 1)}{N^{fe}}\right] \mathbf{L}_z^f. \tag{79}$$

This equation demonstrates that the equilibrium growth rate depends on three key factors. The first is the R&D productivity, θ. Higher θ leads to faster growth, as it increases the efficiency of labor devoted to R&D. Secondly, greater spillovers enhance the productivity of R&D, allowing the economy to grow more rapidly for a given level of labor allocation. Finally, the growth rate is directly proportional to the total labor allocated to R&D in equilibrium, reflecting the central role of innovation in driving economic expansion.

Equilibrium welfare, W^f, is similarly derived by substituting the equilibrium values of $N = N^{fe}$, $E = E^f$, and $r = r^f = \rho$ in Equation (32). Accordingly,

$$W(N^{fe}, E^f, r^f) = N^{fe} \cdot \frac{q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N^{fe}} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}} \cdot \frac{1}{\varepsilon - 1} \left[\frac{(N^{fe} - 1)(\varepsilon - 1)}{N^{fe}\varepsilon - (\varepsilon - 1)} \right]^{(\varepsilon-1)}$$

$$+ \frac{N^{fe}}{N^{fe}\varepsilon - (\varepsilon - 1)} - \theta \cdot \frac{(\varepsilon - 1)[1 + \gamma(N^{fe} - 1)]}{\gamma[N^{fe}\varepsilon - (\varepsilon - 1)]} + \frac{N^{fe} \cdot \rho}{\gamma(N^{fe} - 1)} - N^{fe}\phi.$$

The first term in the expression represents the equilibrium consumer surplus derived from greater product variety and reduced markups. The subsequent terms capture the revenue generated by firms, costs associated with R&D, and fixed costs. Together, these terms provide a comprehensive measure of welfare in the free entry and exit regime.

In the free entry and exit regime, producer surplus is zero because instantaneous profits are driven to zero by free entry. This occurs as the value of a firm, V, equals zero under free entry conditions, implying that firms just cover their costs without making any economic profits. As a result, welfare in this regime is entirely determined by consumer surplus. Consequently, Equation (63) would now reduce to

$$\frac{\partial W}{\partial N} = \frac{\partial CS}{\partial N},$$

and from Equation (48) we know that this is unambiguously positive. which implies that aggregate welfare increases unambiguously with an increase in the number of firms, N. This highlights the central role of consumer surplus in driving welfare in the free entry and exit regime, where producer surplus does not contribute to the aggregate welfare.

This completes the analysis of equilibrium growth and welfare under the free entry and exit regime, which serves as a benchmark for evaluating growth and welfare outcomes across alternative market structures. Building on this foundation, we now introduce the role of the policymaker in our model. Recall that we model two distinct policymaker regimes: the purely benevolent policymaker regime, where the policymaker seeks to maximize aggregate welfare, and the politically motivated policymaker regime, where the policymaker balances competing interests with a bias toward producer surplus. We begin by analyzing the purely benevolent policymaker regime.

6 The Purely Benevolent Policymaker

In our model, the purely benevolent policymaker seeks to maximize overall welfare, defined as the sum of consumer surplus (CS) and producer surplus (Π). Formally, this objective can be expressed as

$$\max_{N} \; \Omega^B(N, E, r) = \int_t^{\infty} \Big[CS(N, E, r) + \Pi(N, E, r) \Big] e^{-\rho(\tau - t)} d\tau,$$

where $\rho > 0$ is the discount rate. The integrand captures the flow of welfare contributions over time. The inclusion of the discount factor ρ reflects the policymaker's valuation of future welfare relative to current welfare.

Although the above formulation appears intertemporal, it simplifies to a static framework under the assumption of steady-state equilibrium. In this steady-state, the contributions to welfare from CS and Π are constant over time. As a result, the integral collapses into a simple scaling factor, reducing the objective function to[8]

$$\max_{N} \; \Omega^B(N, E, r) = \frac{CS(N, E, r) + \Pi(N, E, r)}{\rho}.$$

[8]This simplification arises because, for any constant value x, the integral $\int_t^{\infty} x e^{-\rho(\tau - t)} d\tau$ evaluates to x/ρ. If the analysis were not constrained to the steady-state equilibrium, the integral would retain its intertemporal nature. In this alternative framework, the policymaker would need to consider dynamic transitions in $CS(N, E, r)$ and $\Pi(N, E, r)$ over time, as well as their associated rates of change. This would introduce dynamic terms such as $\dot{CS}(N, E, r)$ and $\dot{\Pi}(N, E, r)$ into the optimization problem, requiring a full characterization of transitional dynamics. Such an approach would make the model analytically intractable for deriving explicit solutions and impede direct comparisons across regimes. Evaluating at steady-state values aligns with the focus of this study, which is to compare long-term outcomes across regimes. Since the free entry and exit regime is inherently analyzed at steady-state equilibrium, maintaining this framework for the policymaker regimes ensures consistency in the comparisons, particularly regarding growth and welfare outcomes.

Thus, maximizing Ω^B is equivalent to maximizing the numerator, $CS + \Pi$, and the discount rate ρ becomes irrelevant to the optimization itself.

The incorporation of the discount factor aligns the policymaker's objective with a time-consistent valuation of welfare, reflecting the reality that future welfare is often valued less than immediate welfare.

In this setting, the static nature of the framework reflects the assumption of a short-sighted policymaker who prioritizes welfare within their immediate term of office. This assumption captures the political economy reality where policymakers often focus on short-term outcomes.

Júlio (2014) adopts a similar approach by formulating the policymaker's utility function with intertemporal components but simplifying the analysis by assuming steady-state equilibrium. Specifically, Júlio introduces two components in the policymaker's utility: the representative household's discounted welfare and steady-state contributions from lobbying groups. Júlio's steady-state assumption is justified on the following grounds. The model assumes that the intertemporal patterns of consumption and contributions are constant and therefore do not affect decision-making at any particular point in time. By focusing on steady-state outcomes, the model avoids complex transitional dynamics while maintaining analytical rigor. The model reflects a realistic political economy setup, where the policymaker balances voter welfare and contributions within their time horizon[9].

[9] In Júlio (2014), the policymaker's utility function integrates both voter welfare and campaign contributions. The framework combines dynamic elements with a steady-state assumption, providing a structured approach to analyzing the policymaker's decisions in the presence of lobbying activities. The policymaker's utility at any point in time is given by

$$u^{pol}(t) = (1-\lambda)\int_t^{\infty} \log(C(\tau)) \cdot e^{-\rho(\tau-t)} d\tau + \lambda \int_t^{\infty} \Psi(\tau) \cdot e^{-\rho(\tau-t)} d\tau.$$

Contributions, $\Psi(\tau)$, are assumed to be steady over time ($\Psi(\tau) = \Psi$), simplifying the dynamic component of the analysis. The discount factor, $\rho > 0$, reflects the policymaker's temporal preferences, emphasizing the present value of welfare and contributions.

While the utility function appears dynamic, Júlio simplifies the analysis by focusing on steady-state outcomes. He assumes that the intertemporal pattern of contributions and consumption is irrelevant, as agents care only about their present values. This assumption allows the utility function to be expressed in steady-state terms, removing transitional dynamics.

We now proceed to characterize the industry equilibrium in the purely benevolent policymaker regime.

6.1 Industry Equilibrium - Purely Benevolent Policymaker Regime

The industry equilibrium is determined by the policymaker, who directly chooses the optimal number of firms, N, to maximize pure social welfare. Formally, the optimization problem is expressed as

$$\underset{N}{max}\ \Omega^B(N,E,r) = \frac{1}{\rho}\Bigg[CS\,(N,E,r) + \Pi(N,E,r)\Bigg], \tag{80}$$

where Ω^B represents total welfare, which is the sum of consumer surplus CS and producer surplus Π. Recall that Assumption 2 ensures that the welfare function is strictly concave in N. This assumption ensures that the first-order condition derived from the maximization exercise in Equation (80) is sufficient to characterize the equilibrium market structure, given E and r.

Substituting for CS and Π from Equation (37) and Equation (55) respectively, the objective becomes

$$\underset{N}{max}\ \frac{1}{\rho}\Bigg[\Bigg\{NE\cdot\frac{q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}}\cdot\frac{1}{\varepsilon-1}\left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{(\varepsilon-1)}\Bigg\}$$

$$+\Bigg\{\frac{NE}{N\varepsilon-(\varepsilon-1)} - E\theta\cdot\frac{(\varepsilon-1)[1+\gamma(N-1)]}{\gamma[N\varepsilon-(\varepsilon-1)]} + \frac{N\cdot r}{\gamma(N-1)} - N\phi\Bigg\}\Bigg]. \tag{81}$$

Note that the consumer surplus and producer surplus are functions of the number of varieties N, expenditure E, and the interest rate, r. The purely benevolent policymaker determines the number of varieties N while taking E and r as given. This decision is akin to issuing R&D licenses to firms, where N represents the number of varieties or firms.

The steady-state simplification results in

$$u^{pol} = (1-\lambda)\cdot\frac{\log(C)}{\rho} + \lambda\cdot\frac{\Psi}{\rho}.$$

This form highlights the balance between voter welfare and campaign contributions, weighted by $(1-\lambda)$ and λ, respectively.

Accordingly, the first order condition for this maximizing exercise would be

$$\frac{\partial \Omega^B}{\partial N} = \frac{\partial CS}{\partial N} + \frac{\partial \Pi}{\partial N} = 0. \tag{82}$$

Using Equation (48) and Equation (59), we expand the first-order condition as

$$\frac{E}{N}\left[\frac{1}{(\varepsilon-1)} + \frac{N}{(N-1)(N\varepsilon-(\varepsilon-1))}\right] - \frac{E(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right] - \frac{r}{\gamma(N-1)^2} - \phi = 0, \tag{83}$$

or, equivalently,

$$\underbrace{E\left[\frac{1}{N(\varepsilon-1)} + \frac{1}{(N-1)(N\varepsilon-(\varepsilon-1))}\right]}_{(a)} \underbrace{- \frac{E(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right] - \frac{r}{\gamma(N-1)^2}}_{(b)} = \phi. \tag{84}$$

Equation (84) determines the equilibrium number of firms, $N^B = N^B(E, r)$, in the purely benevolent policymaker case, given expenditures, E, and the interest rate, r. Notice that there are two terms on the left-hand side of Equation (84). $Term$ (a) is the change in consumer surplus due to a change in N. An increase in N enhances consumer surplus by providing greater product variety and reducing markups through heightened competition. $Term$ (b) is the change in contribution margin due to a change in N. This term accounts for the decline in producer profitability as N increases. It includes two components. The first component reflects the erosion of markups due to intensified competition, reduced by spillovers, γ, and R&D productivity, θ. The second component reflects the rising marginal cost of financing R&D investments. ϕ on the right-hand side is the average fixed cost. It serves as the cost of accommodating an additional firm in the economy.

Thus, the left-hand side of Equation (84) is the net benefit accruing to the economy due to a change in N, and the right-hand side is the associated cost. Equation (84) highlights the trade-offs inherent in determining N^B. A higher N benefits consumers but reduces producer profitability. The planner balances these effects to ensure that the net welfare contribution from entry justifies the associated fixed costs.

At low levels of N, consumer surplus increases rapidly with N due to significant benefits from variety and competition, while producer surplus is relatively stable. As N increases further, the benefits of variety diminish, and the marginal cost of accommodating additional firms rises, leading to a slowdown in welfare gains.

The purely benevolent policymaker regime addresses the inefficiencies of the free entry and exit regime by internalizing R&D spillovers and accounting for the fixed costs of entry. While the free entry and exit regime results in N^{fe}, determined solely by the zero-profit condition, the planner's optimal N^B explicitly balances the trade-offs between consumer surplus and producer surplus. This comparison underscores the efficiency of policy interventions in achieving socially optimal outcomes.

Note that $\lim_{N\to 1}$ of the left-hand side in Equation (84) is indeterminate. This reflects the infeasibility of having a single firm sustain the entire economy. On the other hand, $\lim_{N\to\infty}$ of the left-hand side in Equation (84) is 0, indicating that excessive entry leads to vanishing net welfare gains due to rising fixed costs and reduced profitability.
[Refer to Appendix 5 for the proof of these limits.]

We make a brief note here of certain pertinent issues, which will be discussed later in greater detail in Section 9. The equilibrium number of firms in the purely benevolent policymaker regime, N^B, is expected to be lower than the number of firms in the free entry and exit regime, N^{fe}. The reason for this lies in the purely benevolent policymaker's objective to internalize the fixed costs and the diminishing returns associated with excessive entry, which the free-entry mechanism does not account for. However, this expectation hinges on certain conditions, as established in Appendix 7 (which we discuss in Section 9). Specifically, a sufficient condition for $N^B < N^{fe}$ is established, highlighting the complex interplay between parameters such as γ (R&D spillovers), ϕ (fixed costs), ε (substitution elasticity), and ρ (interest rate). Interestingly, when calibrating the model with specific parameter values (discussed in Section 9), we get a result $N^B > N^{fe}$, which runs counter to the theoretical intuition.

Having discussed the purely benevolent policymaker regime, we now move to the next regime in our model, *viz.,* the politically motivated policymaker regime.

7 The Politically Motivated Policymaker

In this regime, the policymaker seeks to maximize a weighted welfare function that places a disproportionate emphasis on producer surplus. Recognizing the political importance of financial contributions and lobbying, the policymaker assigns a weight $\Lambda > 1$ to producer surplus, Π, relative to consumer surplus, CS. This framework captures the influence of political motivations on the policymaker's decision-making process. Although these political contributions are not modeled explicitly, Λ represents the policymaker's implicit preference for producer surplus over consumer welfare.

Similar to the purely benevolent policymaker regime, this analysis is constrained to the steady-state equilibrium. This effectively static framework assumes that the contributions to welfare from CS and Π remain constant over time, allowing us to abstract from transitional dynamics and focus on the long-term equilibrium outcomes of this regime[10]. Formally, the policymaker's objective function in the politically motivated regime can be expressed as

$$\max_N \ \Omega^P(N, E, r) = \int_t^\infty \Big[CS(N, E, r) + \Lambda \cdot \Pi(N, E, r)\Big] e^{-\rho(\tau - t)} d\tau,$$

where $\rho > 0$ is the discount rate. As in the purely benevolent policymaker regime, the assumption of steady-state equilibrium simplifies this intertemporal optimization problem into a static framework

$$\max_N \ \Omega^B(N, E, r) = \frac{CS(N, E, r) + \Lambda \cdot \Pi(N, E, r)}{\rho}.$$

Maximizing Ω^P is therefore equivalent to maximizing the numerator, $CS(N, E, r) + \Lambda \cdot \Pi(N, E, r)$. By adopting this static framework, we align the analysis of the politically motivated policymaker regime with that of the purely benevolent policymaker regime, facilitating a consistent comparison of long-term outcomes across these regimes.

We now proceed to characterize the industry equilibrium under this regime.

7.1 Industry Equilibrium

The industry equilibrium is determined by the policymaker choosing the optimal number of firms, N^P, to maximize the weighted welfare function. Formally, the optimization problem

[10] Through the imposition of suitable regularity conditions, convergence to the steady state can be ensured.

is given by

$$\underset{N}{max}\ \Omega^P(N,E,r) = \frac{1}{\rho}\Bigg[CS\,(N,E,r) + \Lambda \cdot \Pi(N,E,r)\Bigg], \tag{85}$$

where Ω^P represents the objective function of the politically motivated policymaker. From earlier analysis, we know that $CS(N,E,r)$ is concave ($SOC < 0$) and $\Pi(N,E,r)$ is convex ($SOC > 0$). Assumption 2 ensures that the combined welfare function, $CS + \Pi$, is concave ($SOC < 0$). Since $\Lambda > 1$ is a constant scaling factor for $\Pi(N,E,r)$, it does not alter the curvature of the combined function. Thus, problem $max_N\ \Omega^P = \frac{1}{\rho}[CS + \Lambda\Pi]$ is concave, ensuring that the first-order condition of the maximization problem is sufficient to determine the equilibrium N^P, given E and r.

Substituting for CS and Π from Equation (37) and Equation (55), the maximization exercise can be framed as

$$\underset{N}{max}\ \Bigg[\Bigg\{NE \cdot \frac{q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}} \cdot \frac{1}{\varepsilon-1}\left[\frac{(N-1)(\varepsilon-1)}{N\varepsilon-(\varepsilon-1)}\right]^{(\varepsilon-1)}\Bigg\}$$

$$+\Lambda\left\{\frac{NE}{N\varepsilon-(\varepsilon-1)} - E\theta \cdot \frac{(\varepsilon-1)[1+\gamma(N-1)]}{\gamma[N\varepsilon-(\varepsilon-1)]} + \frac{N\cdot r}{\gamma(N-1)} - N\phi\right\}\Bigg]. \tag{86}$$

Once again, this optimization decision can be interpreted as the policymaker issuing R&D licenses to firms, thereby controlling market entry.

The first-order condition for this maximizing exercise would be

$$\frac{\partial\Omega^P}{\partial N} = \Lambda^{-1}\frac{\partial CS}{\partial N} + \frac{\partial \Pi}{\partial N} = 0. \tag{87}$$

Using the expressions for $\frac{\partial CS}{\partial N}$ and $\frac{\partial \Pi}{\partial N}$ from Equation (47) and Equation (59) respectively, the first-order condition expands to

$$\Lambda^{-1}\frac{E}{N}\left[\frac{1}{(\varepsilon-1)} + \frac{N}{(N-1)(N\varepsilon-(\varepsilon-1))}\right] - \frac{E(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]$$

$$- \frac{r}{\gamma(N-1)^2} - \phi = 0, \tag{88}$$

which can also be written as

$$\underbrace{\Lambda^{-1}E\left[\frac{1}{N(\varepsilon-1)}+\frac{1}{(N-1)(N\varepsilon-(\varepsilon-1))}\right]}_{(a)} \\ -\underbrace{\frac{E(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]-\frac{r}{\gamma(N-1)^2}}_{(b)}=\phi \quad (89)$$

Equation (89) determines the equilibrium number of firms, $N^P = N^P(E, r, \Lambda)$, in the politically motivated policymaker regime, given expenditures, E, and the interest rate, r and the policymaker's weight on the aggregate profits, Λ. Notice that there are two terms on the left-hand side of Equation (89). Term (a) represents the change in consumer surplus resulting from a change in N, reflecting the benefits of greater variety and competition. Term (b) is the change in contribution margin due to a change in N, accounting for the erosion of markups and rising costs associated with additional entry. On the right-hand side, ϕ represents the average fixed cost per firm.

Together, Equation (89) highlights the planner's decision-making process. The planner sets N^P at a point where the net benefit accruing to the economy due to a change in N equals the net cost incurred in the process.

Observe that as $\lim_{N\to 1}$, the left-hand side of Equation (89) becomes indeterminate. This reflects the infeasibility of sustaining an industry with a single firm due to the lack of sufficient competition and variety. On the other hand, as $\lim_{N\to\infty}$, the left-hand side of Equation (89) approaches 0. This indicates that excessive entry leads to vanishing net welfare gains, driven by rising fixed costs and declining profitability for firms.
[Refer to Appendix 6 for the proof of these limits.]

In Section 9, we will examine the relative positions of N^{fe}, N^B and N^P, both analytically and through a numerical calibration exercise.

Having defined and constructed the industry equilibrium for both the purely benevolent and politically motivated policymaker regimes, we now proceed to examine the general equilibrium under these regimes.

8 General Equilibrium - Purely Benevolent and Politically Motivated Policymaker Regimes

In the general equilibrium framework, we account for the feedback between labor market clearing conditions and aggregate expenditures, E. While the number of firms in the industry equilibrium is determined by the policymaker's respective optimization problem for given E and r, general equilibrium requires that the labor market clears, ensuring consistency across all agents in the economy.

The labor market clearing condition ensures that the total demand for labor equals the total labor supply, normalized to unity. Labor demand comes from two sources. The first is for production, denoted by L_X, and the second is R&D investments, denoted by $\mathbf{L}_z$. Formally, the labor market clearing condition is

$$N^i(\Lambda, E^i, \rho)\, L_X(N^i(\Lambda, E^i, \rho), E^i) + \mathbf{L}_z(N^i(\Lambda, E^i, \rho), E^i, \rho) = 1, \quad i \in \{B, P\}, \tag{90}$$

where N^i is the number of firms determined by policymaker as given by Equation (84) and Equation (89) for the purely benevolent and politically motivated regimes, respectively. Note that Equation (84) is a special case of Equation (89) where $\Lambda = 1$.

In the general equilibrium, E adjusts to ensure labor market clearing. However, in the current model, with wages—which are normalized to one—being the only source of income, E does not depend on N. Consequently, the adjustments in the labor market affect the distribution of labor across production and R&D activities but do not influence the equilibrium number of firms, N^i, determined in the industry equilibrium.

Given that E is constant, the N^i determined in the industry equilibrium directly satisfies the general equilibrium conditions. This is because the normalization of $E = w = 1$ removes the potential feedback loop between E and N^i. Thus, the equilibrium number of firms is the same in both the industry-level partial equilibrium and the overall general equilibrium. Although N^i remains fixed, the labor allocation across production, L_X, and R&D, $\mathbf{L}_z$ adjusts to satisfy the labor market clearing condition.

The assumption of a constant E due to normalization of wages simplifies the analysis by ensuring that expenditures do not evolve over time. Consequently, the growth rate of expenditures is zero

$$\frac{\dot{E}^i}{E^i} = 0.$$

This implies that the interest rate, r^i, equals the discount rate, ρ.

$$\frac{\dot{E}^i}{E^i} = r^i - \rho = 0 \implies r^i = \rho.$$

As a result, the number of firms N^i, determined in the industry equilibrium for given $E = 1$ and $r = \rho$, directly satisfies the general equilibrium conditions without requiring further adjustments. Consequently, Equation (84) which specifies the condition for determining the number of firms N under the purely benevolent policymaker regime, can be reformulated as

$$\left[\frac{1}{N^B(\varepsilon-1)} + \frac{1}{(N^B-1)(N^B\varepsilon-(\varepsilon-1))}\right] - \frac{(\varepsilon-1)}{(N^B\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right] - \frac{\rho}{\gamma(N^B-1)^2} = \phi. \quad (91)$$

And similarly, the expression for determining the number of firms under the politically motivated policymaker regime, given by Equation (89), is reformulated as

$$\Lambda^{-1}\left[\frac{1}{N^P(\varepsilon-1)} + \frac{1}{(N^P-1)(N^P\varepsilon-(\varepsilon-1))}\right] - \frac{(\varepsilon-1)}{(N^P\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right] - \frac{\rho}{\gamma(N^P-1)^2} = \phi \quad (92)$$

Having established the general equilibrium under the two policymaker regimes, we summarize the key outcomes. The equilibrium number of firms in the free entry and exit regime is determined by Equation (78). Similarly, the equilibrium number of firms under the purely benevolent and politically motivated policymaker regimes are governed by Equation (91) and Equation (92), respectively. We now proceed to analyze the relative positioning of the N's determined under each of these regimes. With these equilibrium conditions in place, we now turn to a detailed analysis of the relative positioning of the equilibrium number of firms, N, across these three regimes.

9 Determining the Number of Firms

As discussed earlier, the comparison of growth and welfare across the three regimes—free entry and exit, purely benevolent policymaker, and politically motivated policymaker—requires determining the number of firms, N, in each regime. Recall from Equation (30) and Equation (62) that both growth and welfare are functions of N. Consequently, the equilibrium number of firms, N, serves as the foundational variable for understanding the differences in growth rates and welfare outcomes across regimes.

The conditions that determine N under each regime are captured by the following equations: Equation (78) for the free entry and exit regime, Equation (91) for the purely benevolent policymaker regime, and Equation (92) for the politically motivated policymaker regime. Once the equilibrium N is determined for each regime, its value is substituted into the respective growth and welfare equations, enabling a direct comparison across the regimes. This section examines the expected relative positions of N across the regimes, draws insights from theoretical expectations, and introduces a calibration exercise as a practical approach to refine our understanding.

9.1 Free Entry and Exit vs. Purely Benevolent Policymaker Regime

We begin with a comparison of the equilibrium number of firms in the free entry and exit and the purely benevolent policymaker regime. As noted in Subsection 6.1, the number of firms under the free entry and exit regime, N^{fe}, is expected to exceed that in the purely benevolent policymaker regime, N^B. This expectation arises from the contrasting mechanisms governing firm entry. In the free entry and exit regime, firm entry continues until the zero-profit condition is satisfied. This mechanism does not internalize the fixed costs of entry, ϕ, the diminishing returns from excessive entry, and spillovers, γ. Consequently, firms continue to enter the market as long as their revenues cover their fixed and variable costs, potentially resulting in excessive entry relative to the socially optimal level.

Conversely, the purely benevolent policymaker explicitly balances consumer surplus and producer surplus, internalizing the externalities associated with R&D spillovers and accounting for fixed costs. This results in a lower equilibrium number of firms, N^B, compared to N^{fe}, as the policymaker seeks to maximize total welfare rather than allowing unrestricted entry.

While the expectation that $N^{fe} > N^B$ aligns with economic intuition, determining whether this holds always or under specific conditions requires careful examination. We undertake an analytical comparison between N^{fe} as given by Equation (78) and N^B given by Equation (91) in Appendix 7. This leads us to the following result.

Proposition 1 ***The number of firms under the free entry and exit regime is greater than the number of firms under the purely benevolent policymaker regime if the marginal reduction in producer surplus due to entry exceeds twice the fixed costs incurred by firms. Formally,*** $N^{fe} > N^B$ ***if and only if***

$$2\phi \leq -\frac{\partial \Pi}{\partial N}.$$

Proof. In Appendix 7, it has been shown that on comparing Equation (78) and Equation (91), the following necessary condition needs to hold for $N^{fe} > N^B$

$$\phi \leq \left[\frac{(\varepsilon - 1)}{(N\varepsilon - (\varepsilon - 1))^2}\left[1 - \frac{\theta}{\gamma} \cdot (\varepsilon - \gamma)\right] + \frac{\rho}{\gamma(N-1)^2}\right].$$

Using Equation (59) for $\frac{\partial \Pi}{\partial N}$, we know

$$\frac{\partial \Pi}{\partial N} = -\frac{E(\varepsilon - 1)}{(N\varepsilon - (\varepsilon - 1))^2}\left[1 - \frac{\theta}{\gamma} \cdot (\varepsilon - \gamma)\right] - \frac{r}{\gamma(N-1)^2} - \phi,$$

Under the general equilibrium conditions where $E = 1$ and $r = \rho$, this simplifies to

$$\frac{\partial \Pi}{\partial N} = -\frac{\varepsilon - 1}{(N\varepsilon - (\varepsilon - 1))^2}\left[1 - \frac{\theta}{\gamma} \cdot (\varepsilon - \gamma)\right] - \frac{\rho}{\gamma(N-1)^2} - \phi.$$

Rearranging terms, we write

$$\frac{\varepsilon - 1}{(N\varepsilon - (\varepsilon - 1))^2}\left[1 - \frac{\theta}{\gamma} \cdot (\varepsilon - \gamma)\right] + \frac{\rho}{\gamma(N-1)^2} = -\frac{\partial \Pi}{\partial N} - \phi.$$

Substituting this in the necessary condition for $N^{fe} > N^B$ stated above, we obtain

$$\phi \leq -\frac{\partial \Pi}{\partial N} - \phi,$$

which simplifies to

$$2\phi \leq -\frac{\partial \Pi}{\partial N}.$$

This completes the proof.

The condition $2\phi \leq -\frac{\partial \Pi}{\partial N}$ encapsulates the trade-offs between fixed costs and the decline in producer surplus due to entry. The term $-\frac{\partial \Pi}{\partial N}$ reflects the aggregate decline in producer profitability as more firms enter the market. It incorporates three effects. The first is a decline in contribution margins due to intensified competition, captured by the term $-\frac{\varepsilon-1}{(N\varepsilon-(\varepsilon-1))^2}$. The term $-\frac{\rho}{\gamma(N-1)^2}$ captures the higher costs of financing R&D efforts due to the increased competition. The burden of fixed costs is captured by $-\phi$. The negative sign of $\frac{\partial \Pi}{\partial N}$ arises because producer surplus typically decreases as N increases. This reflects the diminishing profitability of firms due to competition, increasing fixed costs, and reduced markups.

Fixed costs represent the entry burden that each additional firm imposes on the industry. The factor 2ϕ acts as a threshold to compare with the aggregate reduction in producer surplus. If this threshold is surpassed, entry under the free entry and exit regime becomes excessive. The factor 2 captures the planner's trade-off between the cost of additional firm entry (represented by fixed costs) and the societal benefits of increased variety and innovation. It ensures that the planner prioritizes the efficient allocation of resources while balancing the welfare of both consumers and producers.

Thus, Proposition 1 states that for $N^{fe} > N^B$ the reduction in producer surplus due to entry, $-\frac{\partial \Pi}{\partial N}$, must be sufficiently large to offset twice the fixed costs. This reflects the planner's role in balancing welfare: if the marginal reduction in producer surplus is not large enough relative to fixed costs, the planner would allow fewer firms to operate, resulting in $N^B < N^{fe}$.

It is important to note here that if the condition $2\phi \leq -\frac{\partial \Pi}{\partial N}$ does not hold, the necessary condition for $N^{fe} > N^B$ is violated and we cannot analytically rank the equilibrium number of firms across the two regimes.

We now proceed to a comparison between the equilibrium number of firms determined under the purely benevolent and politically motivated policymaker regimes.

9.2 Purely Benevolent Policymaker vs. Politically Motivated Policymaker Regimes

We now analyze the equilibrium number of firms under the purely benevolent and politically motivated policymaker regimes, given by Equation (91) and Equation (92), respectively.

Proposition 2 ***The number of firms under the politically motivated policymaker regime is necessarily lower than the number of firms determined in the purely benevolent policymaker regime. Formally,***

$$N^P < N^B$$

Proof. Consider the first-order conditions for the equilibrium number of firms, N, under the purely benevolent policymaker regime, given by Equation (82)

$$\frac{\partial \Omega^B}{\partial N} = \frac{\partial CS}{\partial N} + \frac{\partial \Pi}{\partial N} = 0,$$

and under the politically motivated policymaker regime, given by Equation (87)

$$\frac{\partial \Omega^P}{\partial N} = \Lambda^{-1} \frac{\partial CS}{\partial N} + \frac{\partial \Pi}{\partial N} = 0.$$

Since $\Lambda > 1$ by definition, the weight assigned to $\frac{\partial CS}{\partial N}$ in Equation (82) is scaled down relative to Equation (87), while the term $\frac{\partial \Pi}{\partial N}$ remains unchanged in both equations. As a result, for the first-order condition given by Equation (92) to hold, the equilibrium number of firms, N^P, must be adjusted downward compared to N^B.

Furthermore, from Equation (60) we know that aggregate industry-wide profits, Π, decreases as N rises. A politically motivated policymaker, who places greater weight on producer surplus ($\Lambda > 1$), will therefore set a lower N to prioritize profitability over consumer surplus.

In other words, the politically motivated policymaker internalizes the trade-off between consumer surplus and producer profitability differently from the purely benevolent policymaker, resulting in $N^P < N^B$. This completes the proof.

This result highlights a key distinction between the two policymaker regimes. While the purely benevolent policymaker balances consumer and producer welfare equally, the politically motivated policymaker skews the balance toward producers. This has significant implications for the market structure and welfare outcomes. A lower N^P compared to N^B implies reduced product variety and potentially higher markups, which favor producers at the expense of consumers. This trade-off reflects the inherent inefficiency introduced by political incentives, which distort the socially optimal outcome to cater to producer interests.

Although it is evident that the number of firms under the politically motivated policymaker regime will always be lower than under the purely benevolent policymaker regime, the relative ranking of N between the free entry and exit regime and the politically motivated policymaker regime remains analytically ambiguous. In the calibration exercise, we explore three scenarios based on the weight assigned by the policymaker to industry profits, resulting in three distinct outcomes for the relative positioning of N^{fe} and N^P.

In summary, if the necessary condition mentioned in Proposition 1 holds, we can clearly rank the number of firms across regimes as $N^{fe} > N^B > N^P$. In the following section, as an alternative approach to the analytical comparison across regimes, we propose to perform a numerical calibration exercise to determine the number of firms, N, determined under each regime.

9.3 A Calibration Exercise

We now undertake some meaningful numerical calibration exercise to know how Equation (78), Equation (91), and Equation (92)—which gives us conditions for determining the equilibrium number of firms under the free entry and exit, purely benevolent policymaker and politically motivated policymaker regimes, respectively—behave under specific parametric values. The parametric values adopted in this exercise are largely drawn from Júlio (2014), which employs values grounded in observed empirical facts for the U.S. economy. For a detailed discussion on the derivation and justification of these parameters, we refer readers to Júlio (2014)[11].

[11] The parameter values in the calibration exercise of Júlio (2014) are curated to align with US economic data, combining empirical evidence, theoretical constraints, and iterative refinement. Key parameters like the interest rate ($r = 7\%$) and growth rate ($g = 2.1\%$) are directly calibrated based on long-term real returns and GDP growth rates, while R&D labor intensity ($L_z = 12\% - 15\%$) reflects observed industry patterns. Indirect parameters, such as the elasticity of substitution ($\varepsilon = 6$) and fixed costs ($\phi = 0.07$), are selected to match empirical estimates of markups and labor overhead costs.

Endogenous parameters, including R&D quality elasticity (θ) and spillovers (γ), are calibrated jointly to replicate observed growth and R&D dynamics. Values of $\theta = 0.18$ and $\gamma = 0.7$ ensure realistic R&D labor shares ($\sim 13.5\%$). This process involves iterative refinement, ensuring equilibrium outcomes like labor market clearing, growth, and innovation remain consistent with theoretical and empirical benchmarks.

The only deviation we make from the parametric values assumed in Júlio (2014) pertains to θ, the elasticity of quality with respect to R&D investment. In our framework, θ is constrained by Assumption 1 which requires that

$$\theta < \frac{\gamma}{\varepsilon - \gamma}.$$

For the assumed parametric values, this condition restricts θ to values below 0.13. Therefore, to adhere to Assumption 1, we adopt $\theta = 0.12$.

Table 2 summarizes the parameter values employed in this calibration exercise.

	Parameter	Value
Elasticity of substitution between varieties	ε	6
Spillovers	γ	0.7
Elasticity of quality w.r.t. R&D investment	θ	0.12
Fixed cost	ϕ	0.07
Discount factor	ρ	0.049

Table 2: Parametric Values Adopted in the Calibration Exercise.

Using these parameter values, we plot the behavior of Equation (78), Equation (91), and Equation (92) under three distinct scenarios, each corresponding to a different value of Λ, which is the weight assigned to aggregate producer surplus by the politically motivated policymaker. Note that $\Lambda = 1$ corresponds to a purely benevolent policymaker, while $\Lambda > 1$ reflects the influence of political motivations. By construction, Λ is constrained to values greater than unity in the politically motivated regime.

In Figure 1, we consider the case where $\Lambda = 1.1$, meaning the policymaker values producer surplus 10% more than consumer surplus in their decision-making, reflecting a modest political bias. Note that the red points on the graph represent the equilibrium numbers of firms under each regime, where the curves intersect the horizontal fixed-cost line, ϕ. It can be seen that in this case

$$N^B > N^P > N^{fe}.$$

This result indicates that, under mild political bias, the purely benevolent policymaker fosters the least market concentration, with the highest number of firms. This is because the

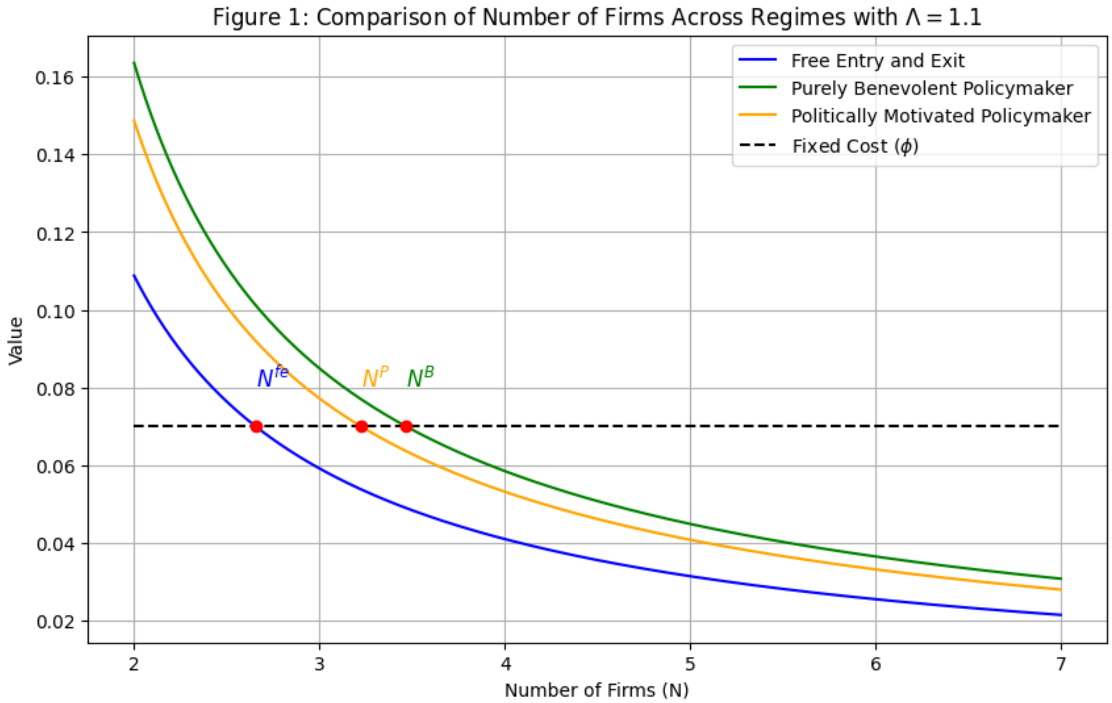

Figure 1: Comparison of Number of Firms Across Regimes, with $\Lambda = 1.1$.

policymaker prioritizes both producer and consumer welfare, allowing for a socially optimal equilibrium that accommodates more firms. The politically motivated policymaker, despite favoring producers slightly, still permits more firms than the free entry and exit regime because the weight placed on consumer welfare, $1/\Lambda = 0.91$, remains substantial.

Notably, the result $N^B > N^{fe}$ challenges theoretical expectations, which as already discussed, suggests N^{fe} should be higher due to unrestricted entry. This counterintuitive outcome can be attributed to the calibration parameters. High spillovers ($\gamma = 0.7$) amplify the welfare benefits of a larger number of firms in the purely benevolent policymaker regime, while relatively low fixed costs ($\phi = 0.7$) do not significantly constrain firm entry. Consequently, the welfare-maximizing behavior of the purely benevolent policymaker encourages more firms than the free entry and exit regime, where diminishing profits naturally limit entry.

In Figure 2, we consider the case where $\Lambda = 1.5$, ndicating that the policymaker values producer surplus 50% more than consumer surplus in their decision-making, reflecting a moderate political bias. The ranking of the equilibrium number of firms changes compared

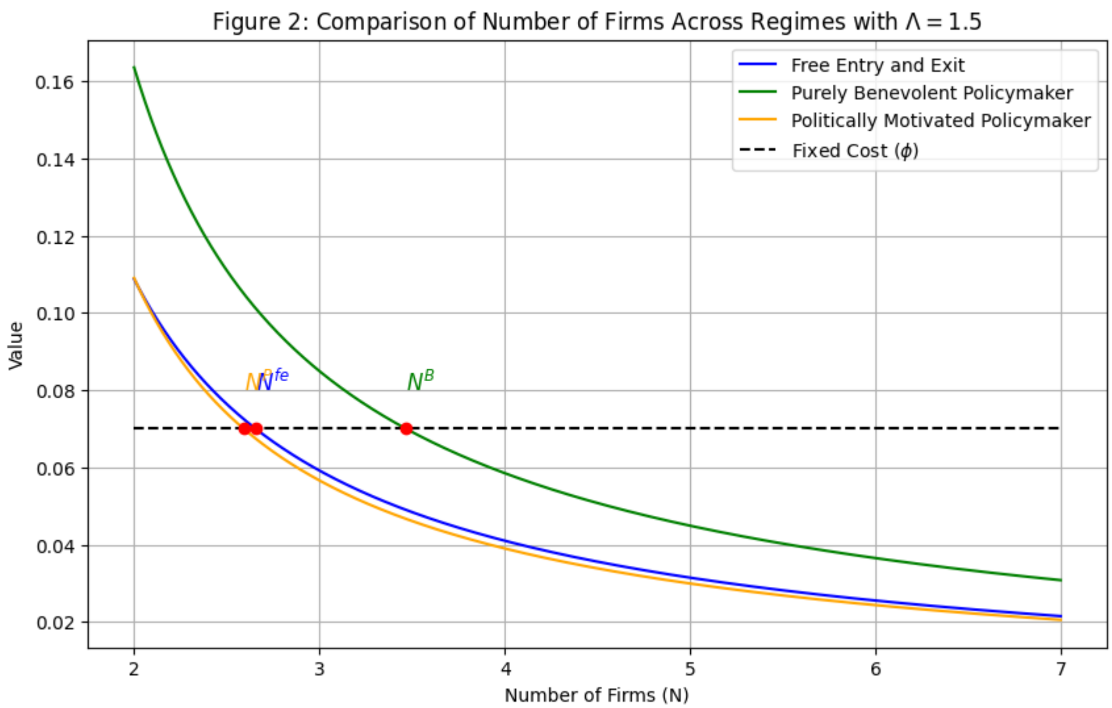

Figure 2: Comparison of Number of Firms Across Regimes, with $\Lambda = 1.5$.

to Figure 1 and is observed as

$$N^B > N^{fe} > N^P.$$

This result suggests that with a more pronounced political bias, the purely benevolent policymaker still fosters the least market concentration, with the highest number of firms. The purely benevolent policymaker's focus on balancing consumer and producer welfare, along with the internalization of spillovers, ensures that the socially optimal number of firms is preserved, maintaining $N^B > N^{fe}$.

The free entry and exit regime, however, now permits more firms than the politically motivated policymaker regime, $N^{fe} > N^P$. This reversal compared to Figure 1 occurs because the higher weight placed on producer surplus ($\Lambda = 1.5$) by the politically motivated policymaker leads to an emphasis on protecting producer profitability. As a result, fewer firms are accommodated to prevent excessive competition that could erode producer surplus.

The ranking $N^B > N^{fe} > N^P$ highlights a shift in market concentration as political bias strengthens. The purely benevolent policymaker ensures the most competitive market structure, while the politically motivated policymaker imposes the most restrictive one. The free entry and exit regime, which neither internalizes spillovers nor balances welfare, now falls between the other two regimes in terms of market concentration. This calibration demonstrates that as the weight on producer surplus increases, market concentration rises under the politically motivated policymaker regime, moving it closer to the restrictive outcomes of a monopolistic market structure.

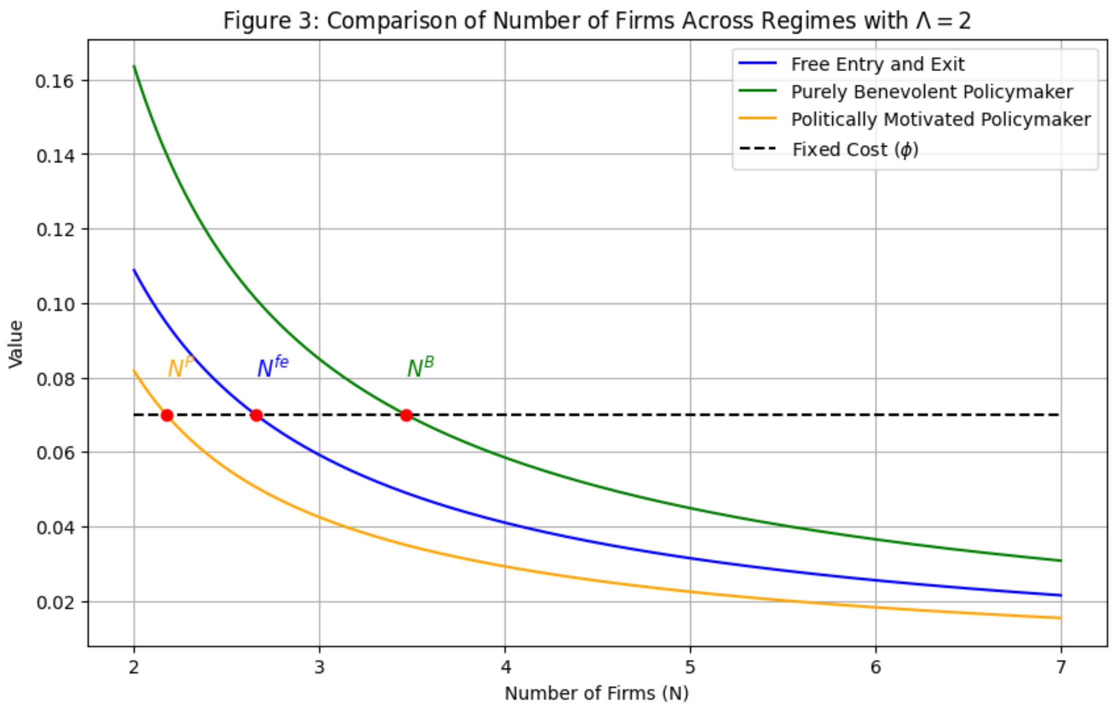

Figure 3: Comparison of Number of Firms Across Regimes, with $\Lambda = 2$.

In Figure 3, we consider the case where $\Lambda = 2$, meaning the policymaker values producer surplus twice as much as consumer surplus in their decision-making, reflecting a strong political bias. The ranking of the equilibrium number of firms is

$$N^B > N^{fe} > N^P.$$

This result reinforces the observation that the purely benevolent policymaker consistently fosters the least market concentration, with the highest number of firms. This outcome occurs because the politically motivated policymaker, heavily favoring producer surplus, restricts firm entry to maximize profitability. The stronger political bias intensifies this tendency, leading to the most concentrated market structure among the three regimes.

The results of the calibration exercise as noticed from Figure 1, Figure 2, and Figure 3, provides several insights into the behavior of the equilibrium number of firms, N, across the three regimes—free entry and exit, purely benevolent policymaker, and politically motivated policymaker—under varying degrees of political bias, Λ. Across all cases, $N^B > N^{fe}$ and $N^B > N^P$. This suggests that the purely benevolent policymaker fosters the least market concentration by allowing the largest number of firms to operate. As Λ increases, the number of firms permitted by the politically motivated policymaker, N^P, decreases significantly. This reflects the policymaker's increasing prioritization of producer surplus over consumer welfare, leading to higher market concentration.

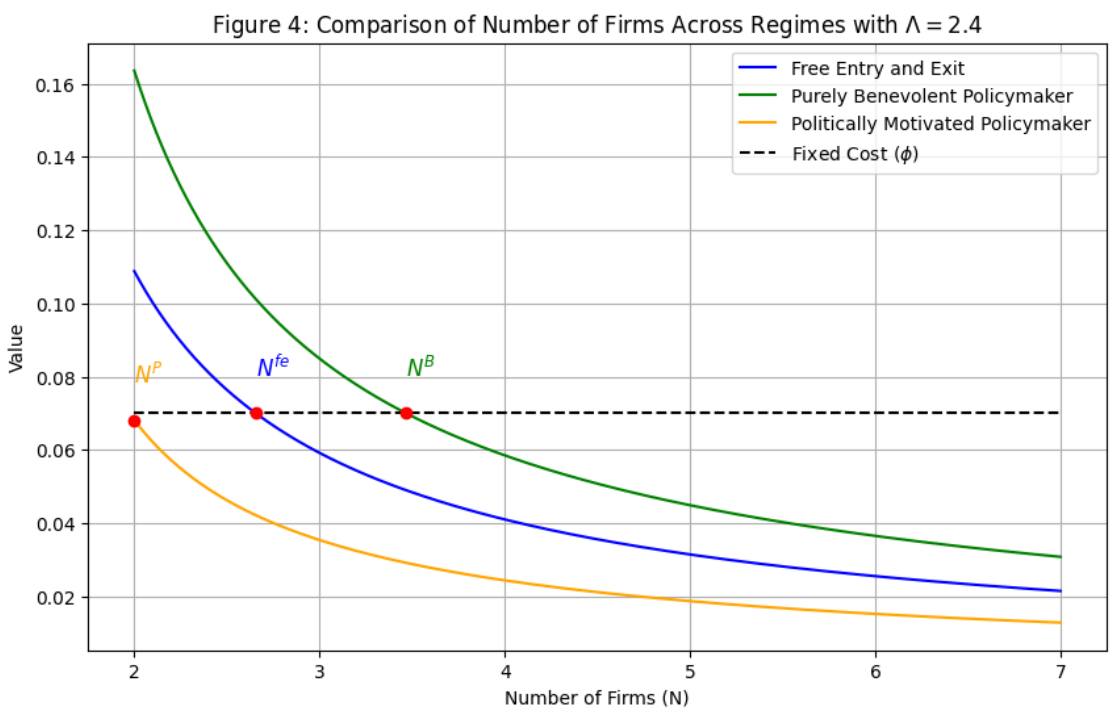

Figure 4: Comparison of Number of Firms Across Regimes, with $\Lambda = 2.4$.

The number of firms under the free entry and exit regime, N^{fe}, remains consistently lower than N^B across all cases. This outcome is theoretically unexpected but can be attributed to the calibration parameters, particularly the high spillovers, γ, which reduce the profitability of entry, discouraging excessive firm entry in this regime.

Our calibration suggests that if Λ increases further, say, $\Lambda = 2.4$, the politically motivated policymaker regime will become infeasible, as the equilibrium number of firms, N^P, may fall below the minimum threshold needed to sustain market operations. This can be observed in Figure 4.

The results suggest that for sufficiently high values of Λ (e.g. $\Lambda = 2.4$), the politically motivated policymaker regime might become infeasible. In such cases, the equilibrium number of firms, N^P, would fall below the minimum threshold required to sustain market operations. In such situations, the policymaker reduces the number of firms in the economy to the absolute minimum feasible number, $N = 2$, which is consistent with the definition of market functionality in our model.

The results from Figure 1, Figure 2, Figure 3, and Figure 4 are formalized as the following three propositions, summarizing the key outcomes of the calibration exercise.

Proposition 3 ***Market concentration is the least in the purely benevolent policymaker regime.***

Proof. The proof is graphical. As already observed, Figure 1, Figure 2, and Figure 3 consistently show that $N^B > N^{fe}$ and $N^B > N^P$. The purely benevolent policymaker internalizes both the fixed costs of entry ϕ, and the spillover effects, γ, enabling a socially optimal number of firms to enter the market. This results in the least concentrated market structure.

Proposition 4 ***Market concentration is lower in the politically motivated policymaker regime compared to the free entry and exit regime under mild political bias. However, as the political bias increases, the politically motivated regime leads to the highest market concentration among the three regimes.***

Proof. Once again, the proof is graphical. It has been seen that at $\Lambda = 1.1$, $N^P > N^{fe}$, showing that mild political bias can still accommodate more firms than the free entry and exit regime. At $\Lambda = 1.5$, N^P drops below N^{fe}, highlighting the restrictive impact of moderate political bias. For higher Λ, N^P is the lowest, with significant market concentration, indicating the inefficiency introduced by strong political bias.

Proposition 5 ***A policymaker with a strong political bias restricts the number of firms to 2.***

Proof. This outcome arises from the fact that as Λ increases, the politically motivated policymaker assigns increasingly higher weight to producer surplus relative to consumer surplus. To maximize producer surplus in such a scenario, the policymaker curtails market competition significantly, leading to higher profit margins for each remaining firm. However, market operations require at least two firms to ensure product variety and prevent a monopoly, which would result in excessive market power and reduced aggregate welfare. Thus, the policymaker stops short of reducing N^P below 2, ensuring a minimal viable market structure while continuing to prioritize producer welfare.

This analysis provides valuable insights into the relationship between policy objectives, market structures, and welfare outcomes, offering a robust framework for evaluating the efficiency of different regulatory regimes.

Building on the results of the equilibrium number of firms across the three regimes, derived both analytically and through the calibration exercise, we now turn our attention to analyzing the implications for growth rates and welfare in this economy, leveraging the insights from the calibration outcomes.

10 Main Results

Having determined the number of firms, N, in each of the three regimes, we now substitute the values of N obtained in each of the regimes and evaluate the growth rate and aggregate welfare. Growth in our economy is given by Equation (30) and welfare is given by Equation (62).

Note the term $\frac{q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}}$ in Equation (62), which pertains to consumer surplus aspect of welfare. It expresses firm i's contribution to the total weighted quality. In symmetry, where all firms have equal quality and price, this fraction becomes $\frac{1}{N}$, reflecting an equal division of total quality among N firms. In symmetric equilibrium, each firm's contribution to aggregate quality (relative quality) matches its contribution to aggregate demand (market share), $\frac{1}{N}$. This is because all firms are identical in prices and qualities, ensuring equal weighting in both dimensions. The equality of relative quality and market share reflects the uniform distribution of demand and quality contributions across all firms. The equality between these terms holds only under the symmetry assumptions. In asymmetric scenarios, where firms differ in price or quality, the two terms will diverge, reflecting the respective demand- and quality-weighted contributions of each firm.

Therefore, the consumer surplus given by Equation (37) is now reduced to

$$CS = \frac{1}{\varepsilon - 1} \left[\frac{(N-1)(\varepsilon - 1)}{N\varepsilon - (\varepsilon - 1)} \right]^{(\varepsilon - 1)}.$$

And the welfare in the economy, as given by Equation (62), now becomes

$$W = \frac{1}{\varepsilon - 1} \left[\frac{(N-1)(\varepsilon - 1)}{N\varepsilon - (\varepsilon - 1)} \right]^{(\varepsilon - 1)} + \frac{N}{N\varepsilon - (\varepsilon - 1)} - \theta \cdot \frac{(\varepsilon - 1)[1 + \gamma(N-1)]}{\gamma[N\varepsilon - (\varepsilon - 1)]} + \frac{N \cdot \rho}{\gamma(N-1)} - N\phi.$$

We use the above two revised expressions while estimating consumer surplus and welfare using the results of the calibration exercise.

In keeping with our calibration exercise, we present the growth rate and welfare in each of the three regimes in Table 3.

Based on the values reported in Table 3, we now present the results obtained through the calibration exercise as the following two propositions, which constitute our main results.

Proposition 6 ***In the purely benevolent policymaker regime***

i The growth rate of the economy is the lowest compared to the free entry and politically motivated policymaker regimes.

ii While consumer surplus is the highest, and producer surplus is the least, the overall welfare is also the lowest here as compared to the other two regimes.

Proof. From Table 3, the growth rate under the purely benevolent policymaker regime is 0.0047, the lowest across the regimes. This is because a higher number of firms ($N = 3.4665$) spreads R&D resources thinly across firms, reducing the intensity of innovation and, consequently, the growth rate.

Regime	N	Growth Rate	Aggregate CS	Industry Profits	Aggregate Welfare
Purely Benevolent Policymaker	3.4665	0.0047	0.0580	-0.0728	-0.0148
Free Entry and Exit	2.6557	0.0056	0.0498	0.0000	0.0498
Politically Motivated Policymaker ($\Lambda = 1.1$)	3.2262	0.0049	0.0560	-0.0524	0.0036
Politically Motivated Policymaker ($\Lambda = 1.5$)	2.5906	0.0057	0.0488	0.0066	0.0554
Politically Motivated Policymaker ($\Lambda \geq 2.4$)	2	0.0070	0.0372	0.0776	0.1147

Table 3: Comparison of Growth Rate and Welfare Across Regimes, and for Different Weights on Profits.

The aggregate consumer surplus is highest at 0.0580, as expected, due to the larger number of firms fostering competition and greater product variety. However, producer surplus is the least (-0.0728), as intense competition erodes profitability. These opposing effects result in the lowest aggregate welfare at -0.0148.

Thus, the purely benevolent policymaker prioritizes consumer welfare but at the cost of producer profitability and innovation, leading to suboptimal aggregate welfare and growth.

Proposition 7 ***In the politically motivated policymaker regime, for moderate political bias, say*** $\Lambda = 1.5$***,***

i The growth rate of the economy is the highest among the three regimes.

ii While consumer surplus is the lowest, and producer surplus is the highest, the overall welfare is also the highest.

For stronger political bias, say, $\Lambda \geq 2.4$***, growth and welfare improve further but with a critical constraint: the number of firms stabilizes at*** $N^P = 2$***, the minimum threshold for a viable market.***

Proof. For $\Lambda = 1.5$, Table 3 shows that the growth rate is 0.0057, higher than both the free entry (0.0057) and purely benevolent (0.0047) regimes. This occurs because the politically motivated policymaker prioritizes producer surplus, ensuring higher profitability for fewer firms, which encourages concentrated R&D investment and accelerates growth.

Aggregate consumer surplus is the lowest at 0.0488 due to fewer firms ($N = 2.5906$), which reduces competition and variety. However, producer surplus is the highest at 0.0066, and the overall welfare (0.0554) exceeds that of the other regimes, balancing the trade-offs effectively.

As discussed earlier in Proposition 5, for $\Lambda \geq 2.4$, the number of firms, N^P, reduces to the minimum viable threshold of 2, enabling the maximum possible growth (0.0070) and aggregate welfare (0.1147).

A Discussion on the Results from the Calibration Exercise

The results of the calibration exercise, as seen in Table 3 and presented graphically in Figure 5.

We observe from the calibration exercise that the consumer surplus and producer surplus behave in line with theoretical expectations. In accordance with Equation (48), the consumer surplus increases as the number of firms rises. This is because a greater number of firms enhances competition, reducing prices, improving product variety and spillovers, and increasing the overall benefits to consumers. The calibration results confirm this trend across all regimes, as N is highest in the purely benevolent policymaker regime, and the corresponding consumer surplus is the maximum at 0.0580.

On the other hand, producer surplus decreases with N, as expected from Equation (60) and Assumption 1. A higher number of firms intensifies competition, leading to reduced markups and profits per firm. This effect is particularly pronounced in the purely benevolent policymaker regime, where the producer surplus is negative (-0.0728). The calibration

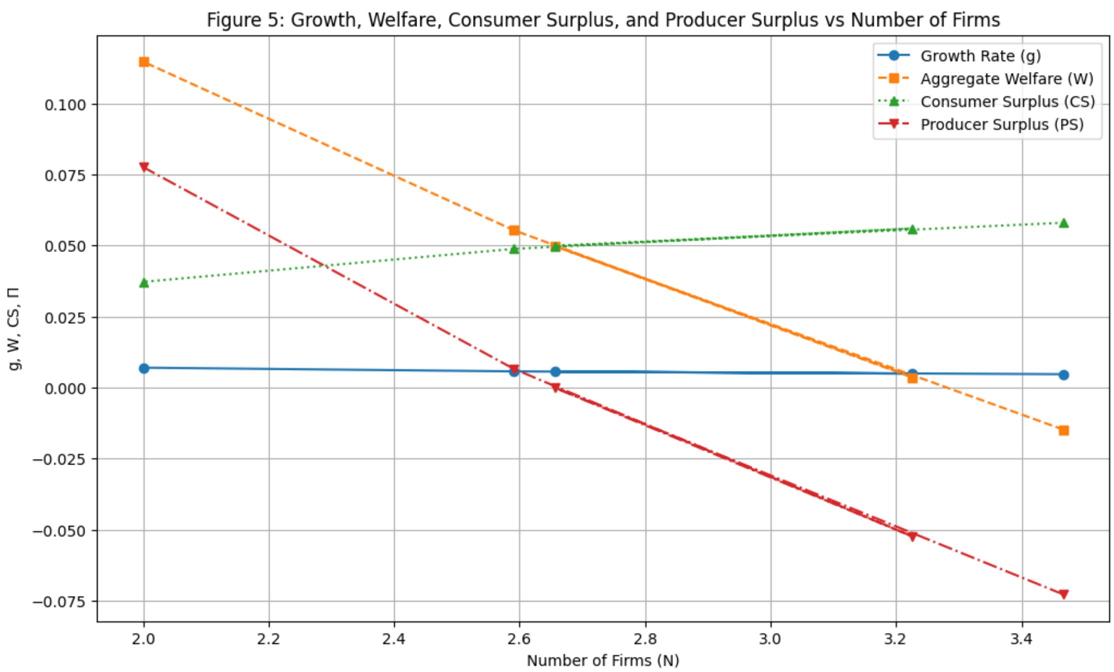

Figure 5: Relationship between N and Growth Rate, Aggregate Welfare, Consumer Surplus, and Producer Surplus.

exercise confirms that producer surplus declines as N increases, and is lowest in regimes with the highest N. Conversely, when N is lower, as in $N = 2$ with $\Lambda \geq 2.4$, firms enjoy larger markups and higher profits.

Thus, the calibration exercise validates the theoretical trade-off between consumer and producer surpluses: while consumers benefit from increased competition, producers incur losses due to diminished market power.

The results on growth rate and aggregate welfare present more nuanced and interesting dynamics, underscoring the intricate trade-offs between innovation, market structure, and welfare outcomes.

The growth rate is primarily driven by R&D investments, which depend on the profitability of firms. When the number of firms is high, R&D resources are spread thinly, reducing the intensity of innovation per firm. Consequently, growth is lower in regimes with larger N, as seen in the purely benevolent policymaker regime ($g = 0.0047$).

In contrast, when N is lower, firms can concentrate R&D investments, leading to higher innovation and faster growth. This is evident in the politically motivated policymaker regime with strong political bias $\Lambda \geq 2.4$, where $N = 2$ results in the highest growth rate ($g = 0.0070$). This aligns with the theoretical prediction that market concentration, to a certain extent, fosters innovation by enabling firms to allocate more resources to R&D.

Aggregate welfare is determined by the combined effects of consumer surplus and producer surplus. While the consumer surplus increases with N, the producer surplus declines. The calibration exercise reveals that, as N rises, the incremental gains in the consumer surplus fail to offset the losses in the producer surplus, resulting in a decline in aggregate welfare. Beyond a certain threshold, the marginal improvements in the consumer surplus are outweighed by the declines in the producer surplus and the negative effects of diluted R&D efforts on growth.

The calibration exercise highlights distinct trade-offs between growth, consumer surplus, and aggregate welfare across regimes. The purely benevolent policymaker achieves the highest consumer surplus but at the expense of producer surplus and growth, resulting in the lowest aggregate welfare. The politically motivated policymaker balances these trade-offs more effectively under moderate political bias, achieving the highest welfare and growth. However, with stronger political bias, the market concentration becomes extreme, which maximizes welfare and growth but risks infeasibility due to insufficient market diversity.

11 Conclusion

This chapter provides a comprehensive analysis of the relationship between market structure, growth, and welfare across three distinct regimes: free entry and exit, purely benevolent policymaker, and politically motivated policymaker. Using a dynamic general equilibrium framework with endogenous R&D, we explore how the number of firms, N, varies under each regime and its implications for economic outcomes.

Our findings reveal that market concentration is least under the purely benevolent policymaker regime, where welfare is maximized by balancing consumer and producer interests. However, this regime also exhibits the lowest growth rates, as reduced profits discourage R&D investment. The free entry and exit regime, governed by zero-profit conditions, results in higher market concentration and moderate welfare outcomes. The politically motivated policymaker regime introduces a unique dynamic, where moderate political bias ($\Lambda = 1.5$) yields the highest growth and welfare, as reduced competition incentivizes innovation without overly suppressing consumer surplus. However, as Λ increases beyond a critical threshold ($\Lambda \geq 2.4$), the equilibrium number of firms falls below the minimum sustainable level, rendering this regime infeasible.

The calibration exercise highlights the trade-offs between consumer and producer surplus in determining aggregate welfare. While consumer surplus increases with N, producer surplus declines due to intensified competition. This interplay leads to an ambiguous welfare trajectory, emphasizing the need for balanced policymaking. Importantly, our results challenge conventional notions by showing that welfare does not always align with growth, as the regime with the highest growth rate (politically motivated policymaker with moderate Λ) does not necessarily exhibit the highest welfare.

We acknowledge that an analytical comparison across regimes would further enhance the rigor of our argument. However, the main results presented in this paper offer an alternative explanation in the wake of the ambiguities presented by the analytical comparison. Our calibration exercise provides a practical and intuitive lens through which the relationship between market structure, growth, and welfare can be understood.

In conclusion, this chapter underscores the critical role of policymaker objectives in shaping market outcomes. By integrating political economy dynamics with market structure analysis, we provide actionable insights for designing policies that foster innovation, sustain competitive markets, and optimize welfare. Future research could extend this framework by explicitly modeling lobbying behaviors and exploring the implications of heterogeneous firm characteristics on growth and welfare.

Appendices

Appendices

Appendix 1: Consumer's maximization problem

Recall that the consumer maximizes

$$\int_t^{\infty} \left[\log \left(\left[\sum_{i=1}^{N} (q_i x_i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \right) e^{-\rho(\tau-t)},\right.$$

subject to $E = \sum_{i=1}^{N} p_i \cdot x_i$. The Lagrangian for the maxmisation exercise would then be

$$\mathcal{L} = \int_t^{\infty} \left[\log \left(\left[\sum_{i=1}^{N} (q_i x_i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \right) e^{-\rho(\tau-t)} - \lambda(\tau) \left(\sum_{i=1}^{N} p_i x_i - E(\tau) \right) \right] d\tau,$$

The first-order condition for this maximization exercise with respect to x_i would be

$$\frac{1}{\left[\sum_{i=1}^{N} (q_i x_i)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}} \cdot \frac{\varepsilon}{\varepsilon-1} \cdot \left[\sum_{i=1}^{N} (q_i x_i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{1}{\varepsilon-1}} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot (q_i x_i)^{-1/\varepsilon} q_i = \lambda p_i. \tag{93}$$

and similarly, the first-order condition with respect to x_j would be

$$\frac{1}{\left[\sum_{i=1}^{N} (q_i x_i)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}} \cdot \frac{\varepsilon}{\varepsilon-1} \cdot \left[\sum_{i=1}^{N} (q_i x_i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{1}{\varepsilon-1}} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot (q_j x_j)^{-1/\varepsilon} q_j = \lambda p_j. \tag{94}$$

Dividing equation Equation (93) by Equation (94) we have

$$\left[\frac{q_i x_i}{q_j x_j}\right]^{-1/\varepsilon} \cdot \frac{q_i}{q_j} = \frac{p_i}{p_j},$$

which can be rewritten as

$$\left[\frac{q_j}{q_i}\right]^{\frac{1-\varepsilon}{\varepsilon}} \left[\frac{x_i}{x_j}\right]^{-1/\varepsilon} = \frac{p_i}{p_j},$$

which can also be expressed as

$$\left[\frac{x_i}{x_j}\right]^{-1/\varepsilon} = \frac{p_i}{p_j} \left[\frac{q_i}{q_j}\right]^{\frac{1-\varepsilon}{\varepsilon}}.$$

Raising the power on both sides by $-\varepsilon$, we have

$$\frac{x_i}{x_j} = \left[\frac{p_i}{p_j}\right]^{-\varepsilon} \left[\frac{q_i}{q_j}\right]^{\varepsilon-1}.$$

On multiplying both sides by $p_i x_j$, we have

$$p_i x_i = p_i x_j \left[\frac{p_i}{p_j}\right]^{-\varepsilon} \left[\frac{q_i}{q_j}\right]^{\varepsilon-1},$$

which further simplifies to

$$p_i \cdot x_i = x_j \cdot p_i^{1-\varepsilon} \cdot p_j^{\varepsilon} \left[\frac{q_i}{q_j}\right]^{\varepsilon-1}.$$

By summing up over i on both sides, we have

$$\sum_i p_i x_i = x_j \cdot p_j^{\varepsilon} \cdot q_j^{-(\varepsilon-1)} \cdot \sum_i p_i^{1-\varepsilon} q_i^{\varepsilon-1}. \tag{95}$$

We know that $E = \sum_i p_i x_i$. Therefore, Equation (95) can be written as

$$E = x_j \cdot p_j^{\varepsilon} \cdot q_j^{-(\varepsilon-1)} \cdot \sum_i p_i^{1-\varepsilon} q_i^{\varepsilon-1}$$

which further simplifies to

$$x_j = \frac{E \cdot p_j^{-\varepsilon} \cdot q_j^{\varepsilon-1}}{\sum_i p_i^{-(\varepsilon-1)} q_i^{\varepsilon-1}}$$

On multiplying and dividing the right-hand side by p_j we obtain

$$x_j = \frac{E \cdot p_j^{-(\varepsilon-1)} \cdot q_j^{\varepsilon-1}}{p_j \sum_i p_i^{-(\varepsilon-1)} q_i^{\varepsilon-1}}$$

Analogously, we will then obtain

$$x_i = \frac{E \cdot p_i^{-(\varepsilon-1)} \cdot q_i^{\varepsilon-1}}{p_i \sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{\varepsilon-1}} \tag{96}$$

We designate the market share captured by firm i by $S(p_i, q_i)$, such that

$$S(p_i, q_i) = \frac{p_i^{-(\varepsilon-1)} \cdot q_i^{\varepsilon-1}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{\varepsilon-1}}$$

Therefore, Equation (96) can now be written as

$$x_i = \frac{E \cdot S(p_i, q_i))}{p_i}.$$

Appendix 2: Maximization of the Hamiltonian with respect to the price, p_i

The Hamiltonion of the firm's maximization problem would be

$$H_i^{cv} = (p_i - 1)\cdot \frac{E p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}}{p_i \sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}} - (L_{z_i} + \phi) + \mu_i \cdot L_{z_i}\left[z_i + \gamma \sum_{j\neq i}^{N} z_j\right]$$

where z_i is the state-variable, μ_i is the co-state variable and p_i and L_{z_i} are the control variables.

The first-order condition in the control variable, that is, $\partial H_i^{cv}/\partial p_i$ would be

$$p_i \cdot \sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)} \Bigg[-(p_i-1)\cdot E \cdot q_i^{(\varepsilon-1)(\varepsilon-1)} p_i^{-(\varepsilon-1)-1} + E \cdot p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)} \cdot 1 \Bigg]$$

$$-(p_i-1)\cdot E \cdot p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)} \cdot \left[\left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}\right]\cdot 1 + p_i \cdot q_i^{(\varepsilon-1)}\Bigg(-(\varepsilon-1) p_i^{-(\varepsilon-1)-1}\Bigg)\right] = 0$$

By collecting like terms and cancelling out E, this can be rewritten as

$$p_i \sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)} \left[-q_i^{(\varepsilon-1)}(p_i-1)(\varepsilon-1)p_i^{-\varepsilon} + p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1}\right]$$
$$-(p_i-1)p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)} \left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)} - p_i q_i^{(\varepsilon-1)}(\varepsilon-1)p_i^{-\varepsilon}\right] = 0$$

which by some rearrangement of terms, would be

$$p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)} \sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)} \Bigg[-(p_i-1)(\varepsilon-1) + p_i \Bigg]$$
$$-(p_i-1)p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)} \left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)} - p_i q_i^{(\varepsilon-1)}(\varepsilon-1)p_i^{-\varepsilon}\right] = 0$$

Upon dividing both sides by $p_i^{-(\varepsilon-1)} \cdot q_i^{(\varepsilon-1)}$, we obtain

$$-\left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}\right] [(\varepsilon-1)(p_i-1)] + p_i \left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}\right]$$

$$-(p_i-1)\left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)} - p_i^{-(\varepsilon-1) q_i^{(\varepsilon-1)}}\right] = 0$$

which can be simplified to

$$-\left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}\right]\left[(\varepsilon-1)(p_i-1)\right] + p_i\left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}\right] - p_i\left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}\right]$$

$$+\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)} + (\varepsilon-1)(p_i-1)\cdot p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)} = 0$$

By cancelling out like terms on the left-hand side and some rearrangement, we have

$$-\left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}\right]\left[(\varepsilon-1)(p_i-1)\right] + \sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}$$

$$= -(\varepsilon-1)(p_i-1)\cdot p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)} \quad (97)$$

which on collecting like terms on the left-hand side, can be rewritten as

$$\left[\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}\right]\left[-(\varepsilon-1)(p_i-1)+1\right] = -(\varepsilon-1)(p_i-1)\cdot p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}$$

which on rearrangement of terms can be written as

$$-(\varepsilon-1)(p_i-1)+1 = \frac{-(\varepsilon-1)(p_i-1)\cdot p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}}$$

Recall that

$$S_i\left(p_i, q_i\right) = \frac{p_i^{-(\varepsilon-1)} q_i^{(\varepsilon-1)}}{\sum_{j=1}^{N} p_j^{-(\varepsilon-1)} q_j^{(\varepsilon-1)}}$$

Therefore, Equation (97) can now be written as

$$-(\varepsilon-1)(p_i-1)+1 = -(\varepsilon-1)(p_i-1)\cdot S_i\left(p_i, q_i\right)$$

which can be rearranged as

$$(\varepsilon-1)(p_i-1) - (\varepsilon-1)(p_i-1)\cdot S_i\left(p_i, q_i\right) = 1$$

which can further be rearranged as

$$(p_i-1)\left[(\varepsilon-1) - S_i\left(p_i, q_i\right)(\varepsilon-1)\right] = 1$$

which can also be written as

$$p_i - 1 = \frac{1}{(\varepsilon - 1) - S_i\left(p_i, q_i\right)(\varepsilon - 1)}$$

which can further be written as

$$p_i = \frac{1}{(\varepsilon - 1) - S_i\left(p_i, q_i\right)(\varepsilon - 1)} + 1$$

By simplifying the right-hand side, we have

$$p_i = \frac{1 + \varepsilon - 1 - S_i\left(p_i, q_i\right)(\varepsilon - 1)}{\varepsilon - S_i\left(p_i, q_i\right)(\varepsilon - 1) - 1} = \frac{\varepsilon - S_i\left(p_i, q_i\right)(\varepsilon - 1)}{\varepsilon - S_i\left(p_i, q_i\right)(\varepsilon - 1) - 1} \tag{98}$$

Recall that $\xi_i = \varepsilon - (\varepsilon - 1) S_i\left(p_i, q_i\right)$. Therefore, Equation (98) can be rewritten as

$$p_i = \frac{\xi_i}{\xi_i - 1}$$

Appendix 3: Shape of the average R&D curve

The proof consists of three steps. In the first step, we first show that $\partial L_z / \partial N > 0$ for small N. In the second step, we show that $\lim_{N \to \infty} \partial L_z / \partial N$ converges to 0. In the final step, we show that $\partial L_z / \partial N < 0$ for large N.

From Equation (20), we know that

$$L_z(N, E, r) = \frac{1}{\gamma}\left[\theta \cdot \zeta(N) \frac{E}{N \xi(N)} \cdot \frac{\sigma(N)}{N-1} - \frac{r}{N-1}\right].$$

Substituting for $\xi(N) = \varepsilon - \frac{(\varepsilon-1)}{N}$, $\zeta(N) = \frac{(\varepsilon-1)(N-1)}{N}$ and $\sigma = [1 + \gamma(N-1)]$ in the above equation, we have,

$$L_z(N, E, r) = \frac{1}{\gamma}\left[\frac{E\theta}{N} \cdot \frac{(\varepsilon-1)(N-1)}{N\varepsilon - (\varepsilon - 1)} \cdot \frac{[1 + \gamma(N-1)]}{N-1} - \frac{r}{N-1}\right],$$

which can be simplified to

$$L_z(N, E, r) = \frac{1}{\gamma}\left[E\theta(\varepsilon - 1) \cdot \frac{[1 + \gamma(N-1)]}{N(N\varepsilon - (\varepsilon - 1))} - \frac{r}{N-1}\right] \tag{99}$$

The partial derivative of L_z, with respect to N, would be

$$\frac{\partial L_z}{\partial N} = \frac{1}{\gamma} \cdot E\theta(\varepsilon - 1)\left[\frac{N(N\varepsilon - (\varepsilon - 1)) \cdot \gamma - [1 + \gamma(N-1)][2N\varepsilon - (\varepsilon - 1)]}{N^2(N\varepsilon - (\varepsilon - 1))^2}\right] + \frac{r}{\gamma} \frac{1}{(N-1)^2},$$

which can simplified to

$$\frac{\partial L_z}{\partial N} = \frac{1}{\gamma}\cdot\frac{\theta(\varepsilon-1)E}{N(N\varepsilon-(\varepsilon-1))}\left[\gamma-\frac{(1+\gamma(N-1))(2N\varepsilon-(\varepsilon-1))}{N(N\varepsilon-(\varepsilon-1))}\right]+\frac{r}{\gamma}\frac{1}{(N-1)^2}. \tag{100}$$

For small N, that is, as N approaches 1, we have,

$$\left.\frac{\partial L_z}{\partial N}\right|_{(N\to 1)} = \frac{1}{\gamma}\cdot\frac{\theta(\varepsilon-1)E}{(1\cdot\varepsilon-(\varepsilon-1))}\left[\gamma-\frac{(1+\gamma(1-1))(2\varepsilon-(\varepsilon-1))}{(1\cdot\varepsilon-(\varepsilon-1))}\right]+\frac{r}{\gamma}\frac{1}{(1-1)^2},$$

which can be simplified into

$$\left.\frac{\partial L_z}{\partial N}\right|_{(N\to 1)} = \frac{1}{\gamma}\cdot\frac{\theta(\varepsilon-1)E}{\varepsilon-\varepsilon+1}\left[\gamma-\frac{(2\varepsilon-\varepsilon+1)}{\varepsilon-\varepsilon+1}\right]+\frac{r}{\gamma}\frac{1}{(0)^2},$$

which then reduces to

$$\left.\frac{\partial L_z}{\partial N}\right|_{(N\to 1)} = \frac{1}{\gamma}\cdot\theta(\varepsilon-1)E(\gamma-\varepsilon-1)+\frac{r}{\gamma}\cdot\infty=+\infty.$$

Thus, for small values of N, $\frac{\partial L_z}{\partial N}$ approaches $+\infty$, which is greater than 0. This completes the first step of our proof.

In the second step, we show that $\lim_{N\to\infty}\partial L_z/\partial N$ converges to 0. Equation Equation (100) can be re-expressed as

$$\frac{\partial L_z}{\partial N} = \frac{\theta(\varepsilon-1)E}{N(N\varepsilon-(\varepsilon-1))} - \frac{\theta(\varepsilon-1)E[(2N\varepsilon-(\varepsilon-1))+\gamma(2N^2\varepsilon-2N\varepsilon-N\varepsilon+\varepsilon+N-1)]}{\gamma[N(N\varepsilon-(\varepsilon-1))]^2}+\frac{r}{\gamma(N-1)^2}.$$

We consider $\lim_{N\to\infty}\partial L_z/\partial N$ and apply l'Hôpital's rule to the middle term in the right-hand side in the above equation and obtain,

$$\left.\frac{\partial L_z}{\partial N}\right|_{(N\to\infty)} = \frac{\theta(\varepsilon-1)E}{\infty(\infty\varepsilon-(\varepsilon-1))} - \lim_{N\to\infty}\frac{\theta(\varepsilon-1)E[2\varepsilon+\gamma(4N\varepsilon-2\varepsilon-\varepsilon+1]}{\gamma 2[N(N\varepsilon-(\varepsilon-1))][2N\varepsilon-\varepsilon+1]}+\frac{r}{\gamma(\infty-1)^2},$$

which upon simplification and further applying l'Hôpital's rule to the middle term gives

$$\left.\frac{\partial L_z}{\partial N}\right|_{(N\to\infty)} = 0-\lim_{N\to\infty}\frac{\theta(\varepsilon-1)E\cdot(4\gamma\varepsilon)}{2\gamma[2N\varepsilon-\varepsilon+1][2N\varepsilon-\varepsilon+1]+2[N(N\varepsilon-(\varepsilon-1))]\cdot 2\varepsilon}+0,$$

and upon evaluating the limit, we obtain

$$\left.\frac{\partial L_z}{\partial N}\right|_{(N\to\infty)} = 0+0+0 = 0.$$

This completes the second step of our proof. In the third step, to show that $\partial L_z/\partial N < 0$ for large N, we first check for the condition under which $\partial L_z/\partial N < 0$ would be negative. From Equation (100), we know that $\partial L_z/\partial N < 0$ when

$$\frac{1}{\gamma}\cdot\frac{\theta(\varepsilon-1)E}{N(N\varepsilon-(\varepsilon-1))}\left[\gamma-\frac{(1+\gamma(N-1))(2N\varepsilon-(\varepsilon-1))}{N(N\varepsilon-(\varepsilon-1))}\right]+\frac{r}{\gamma}\frac{1}{(N-1)^2} < 0,$$

that is to say,

$$-\frac{1}{\gamma}\cdot\frac{\theta(\varepsilon-1)E}{N(N\varepsilon-(\varepsilon-1))}\left[\gamma-\frac{(1+\gamma(N-1))(2N\varepsilon-(\varepsilon-1))}{N(N\varepsilon-(\varepsilon-1))}\right] > \frac{r}{\gamma}\frac{1}{(N-1)^2},$$

which can be re-expressed as,

$$\frac{\theta(\varepsilon-1)E(N-1)}{N(N\varepsilon-(\varepsilon-1))}\left[\frac{[1+\gamma(N-1)](2N\varepsilon-(\varepsilon-1))}{N(N\varepsilon-(\varepsilon-1))}-\gamma\right] > \frac{r}{N-1}. \tag{101}$$

If the above condition holds for large N, then $\partial L_z/\partial N < 0$ for large N. This is what we seek to prove in the third step. The left-hand side of the above the equation simplifies to

$$\frac{\theta(\varepsilon-1)E(N-1)}{N(N\varepsilon-(\varepsilon-1))}\left[\frac{[1+\gamma(N-1)](N\varepsilon+N\varepsilon-(\varepsilon-1))}{N(N\varepsilon-(\varepsilon-1))}-\gamma\right],$$

which by re-arrangement of terms, yields,

$$\frac{\theta(\varepsilon-1)E(N-1)}{N(N\varepsilon-(\varepsilon-1))}\left[\frac{[1+\gamma(N-1)]N\varepsilon}{N(N\varepsilon-(\varepsilon-1))}+\frac{[1+\gamma(N-1)](N\varepsilon-(\varepsilon-1))}{N(N\varepsilon-(\varepsilon-1))}-\gamma\right],$$

which further simplifies to

$$\frac{\theta(\varepsilon-1)E(N-1)}{N(N\varepsilon-(\varepsilon-1))}\left[\frac{[1+\gamma(N-1)]\varepsilon}{(N\varepsilon-(\varepsilon-1))}+\frac{[1+\gamma(N-1)]}{N}-\gamma\right]$$

which can be re-expressed as

$$\frac{\theta(\varepsilon-1)E}{N(N\varepsilon-(\varepsilon-1))} \cdot\left[[1+\gamma(N-1)]\cdot\underbrace{\frac{\varepsilon(N-1)}{N\varepsilon-(\varepsilon-1)}}_{(a)}+[1+\gamma(N-1)]\cdot\underbrace{\frac{(N-1)}{N}}_{(b)}-\gamma(N-1)\right].$$

Note that term (a) can also be written as $\frac{1}{1+\frac{1}{N\varepsilon-\varepsilon}}$ and as $N \to \infty$, it approaches 1. Similarly, term (b) can also be written as $1-\frac{1}{N}$ and as $N \to \infty$, it approaches 1. Therefore, for large N, the above equation converges to

$$\frac{\theta(\varepsilon-1)E}{N(N\varepsilon-(\varepsilon-1))}\Bigg[[1+\gamma(N-1)]\cdot 1+[1+\gamma(N-1)]\cdot 1-\gamma(N-1)\Bigg],$$

which reduces to

$$\frac{\theta(\varepsilon-1)E}{N(N\varepsilon-(\varepsilon-1))}\Bigg[1+\gamma(N-1)+1+\gamma(N-1)-\gamma(N-1)\Bigg],$$

which further reduces to

$$\frac{\theta(\varepsilon-1)E}{N(N\varepsilon-(\varepsilon-1))}(2+\gamma(N-1)).$$

Therefore, the condition in Equation (101) would now reduce to

$$\frac{\theta(\varepsilon-1)E}{N(N\varepsilon-(\varepsilon-1))}(2+\gamma(N-1)) > \frac{r}{N-1}. \tag{102}$$

Recall from equation Equation (99) that

$$L_z(N,E,r)=\frac{1}{\gamma}\left[E\theta(\varepsilon-1)\cdot\frac{[1+\gamma(N-1)]}{N(N\varepsilon-(\varepsilon-1))}-\frac{r}{N-1}\right].$$

Since $L_z > 0$, the above equation would mean that,

$$\frac{1}{\gamma}\left[E\theta(\varepsilon-1)\cdot\frac{[1+\gamma(N-1)]}{N(N\varepsilon-(\varepsilon-1))}-\frac{r}{N-1}\right]>0,$$

which can also be expressed as

$$\frac{\theta(\varepsilon-1)E}{N(N\varepsilon-(\varepsilon-1))}(1+\gamma(N-1)) > \frac{r}{N-1}. \tag{103}$$

On comparing Equation (102) and Equation (103), we see that since Equation (103) holds, Equation (102) also holds true.

This completes the third step in our proof.

Thus, the average R&D investment, L_z is hump-shaped in the number of firms.

Appendix 4: Limits of the left-hand side of Equation (78)

Equation (78) determines the equilibrium number of firms under the free entry and exit regime. Herein, we analyze how the left-hand side of this equation behaves in the limits. Recall Equation (78):

$$\frac{1}{N^{fe}\varepsilon - (\varepsilon - 1)}\left[1 - \frac{\theta}{\gamma} \cdot \frac{(\varepsilon - 1)[1 + \gamma(N^{fe} - 1)])}{N^{fe}}\right] + \frac{\rho}{\gamma \cdot (N^{fe} - 1)} = \phi.$$

As N^{fe} approaches 1, the left-hand side would approach

$$\lim_{N^{fe}\to 1}\frac{1}{N^{fe}\varepsilon - (\varepsilon - 1)}\left[1 - \frac{\theta}{\gamma} \cdot \frac{(\varepsilon - 1)[1 + \gamma(N^{fe} - 1)])}{N^{fe}}\right] + \lim_{N^{fe}\to 1}\frac{\rho}{\gamma \cdot (N^{fe} - 1)}.$$

On evaluating the limit, the above equation yields,

$$\frac{1}{1 \cdot \varepsilon - \varepsilon + 1}\left[1 - \frac{\theta}{\gamma} \cdot \frac{(\varepsilon - 1)[1 + \gamma(1 - 1)]}{1}\right] + \frac{\rho}{\gamma \cdot (1 - 1)},$$

which can be simplified to

$$\left[1 - \frac{\theta}{\gamma} \cdot (\varepsilon - 1)\right] + \frac{\rho}{\gamma \cdot 0}.$$

The above equation yields:

$$\left[1 - \frac{\theta}{\gamma} \cdot (\varepsilon - 1)\right] + \infty,$$

that is to say,

$$\lim_{N^{fe}\to 1}\frac{1}{N^{fe}\varepsilon - (\varepsilon - 1)}\left[1 - \frac{\theta}{\gamma} \cdot \frac{(\varepsilon - 1)[1 + \gamma(N^{fe} - 1)])}{N^{fe}}\right] + \lim_{N^{fe}\to 1}\frac{\rho}{\gamma \cdot (N^{fe} - 1)} = \infty.$$

Thus, as $N \to 1$, the left-hand side of Equation (78) approaches ∞.

Similarly, as N^{fe} approaches ∞, the left-hand side of Equation (78) would approach

$$\lim_{N^{fe}\to \infty}\frac{1}{N^{fe}\varepsilon - (\varepsilon - 1)}\left[1 - \frac{\theta}{\gamma} \cdot \frac{(\varepsilon - 1)[1 + \gamma(N^{fe} - 1)])}{N^{fe}}\right] + \frac{\rho}{\gamma \cdot (N^{fe} - 1)}.$$

On evaluating the limit, the above equation yields,

$$\frac{1}{\infty \cdot \varepsilon - (\varepsilon - 1)}\left[1 - \frac{\theta}{\gamma} \cdot \frac{(\varepsilon - 1)[1 + \gamma(\infty - 1)])}{\infty}\right] + \frac{\rho}{\gamma \cdot (\infty - 1)},$$

which simplified to

$$0 + 0.$$

That is,

$$\lim_{N^{fe}\to\infty} \frac{1}{N^{fe}\varepsilon - (\varepsilon-1)} \left[1 - \frac{\theta}{\gamma} \cdot \frac{(\varepsilon-1)[1+\gamma(N^{fe}-1)])}{N^{fe}}\right] + \frac{\rho}{\gamma \cdot (N^{fe}-1)} = 0.$$

Thus, as $N^{fe} \to \infty$, the left-hand side of Equation (78) approaches 0.

Appendix 5: Limits of the left-hand side of Equation (84)

Equation (84) determines the equilibrium number of firms in the purely benevolent policy-maker regime. Herein, we analyze how the left-hand side of this equation behaves in the limits. Recall Equation (84):

$$E\left[\frac{1}{N(\varepsilon-1)} + \frac{1}{(N-1)(N\varepsilon-(\varepsilon-1)}\right] - \frac{E(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right] - \frac{r}{\gamma(N-1)^2} = \phi.$$

As N approaches 1, the left-hand side of Equation (84) would approach

$$\lim_{N\to 1} E\left[\frac{1}{N(\varepsilon-1)} + \frac{1}{(N-1)(N\varepsilon-(\varepsilon-1)}\right] - \lim_{N\to 1}\frac{E(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right] - \lim_{N\to 1}\frac{r}{\gamma(N-1)^2}.$$

On evaluating the limit, we get

$$E\left[\frac{1}{1\cdot(\varepsilon-1)} + \frac{1}{(1-1)(1\cdot\varepsilon-(\varepsilon-1)}\right] - \frac{E(\varepsilon-1)}{(1\cdot\varepsilon-(\varepsilon-1))^2}\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right] - \frac{r}{\gamma(1-1)^2},$$

which can be reduced to

$$E\left[\frac{1}{(\varepsilon-1)} + \frac{1}{0\cdot(\varepsilon-\varepsilon+1)}\right] - \frac{E(\varepsilon-1)}{(\varepsilon-\varepsilon+1)^2}\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right] - \frac{r}{\gamma(0)^2},$$

which further simplifies to

$$\frac{E}{(\varepsilon-1)} + \infty - E(\varepsilon-1)\cdot\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right] - \infty,$$

which is indeterminate. Thus, as $N \to 1$, limit of the left-hand side of Equation (84) is indeterminate.

As N aprooaches ∞, the left-hand side of Equation (84) approaches

$$\lim_{N\to\infty} E\left[\frac{1}{N(\varepsilon-1)} + \frac{1}{(N-1)(N\varepsilon-(\varepsilon-1))}\right] - \lim_{N\to\infty}\frac{E(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right] - \lim_{N\to\infty}\frac{r}{\gamma(N-1)^2}.$$

On evaluating the limit, we obtain,

$$E\left[\frac{1}{\infty\cdot(\varepsilon-1)} + \frac{1}{(\infty-1)(\infty\cdot\varepsilon-\varepsilon+1)}\right] - \frac{E(\varepsilon-1)}{(\infty\cdot\varepsilon-\varepsilon+1)^2}\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right] - \frac{r}{\gamma(\infty-1)^2},$$

which simplifies to

$$E\left[\frac{1}{\infty}+\frac{1}{\infty}\right] - \frac{E(\varepsilon-1)}{\infty}\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right] - \frac{r}{\gamma\cdot\infty} = 0.$$

Thus, as $N \to \infty$, limit of the left-hand side of Equation (84) is 0.

Appendix 6: Limits of the left-hand side of Equation (89)

Equation (89) determines the equilibrium number of firms in the purely benevolent policy-maker regime. Herein, we analyze how the left-hand side of this equation behaves in the limits. Recall Equation (89):

$$\Lambda^{-1}E\left[\frac{1}{N(\varepsilon-1)} + \frac{1}{(N-1)(N\varepsilon-(\varepsilon-1))}\right] - \frac{E(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right] - \frac{r}{\gamma(N-1)^2} = \phi.$$

As N approaches 1, left-hand side of equation (84) would approach

$$\lim_{N\to 1} \Lambda^{-1}E\left[\frac{1}{N(\varepsilon-1)} + \frac{1}{(N-1)(N\varepsilon-(\varepsilon-1))}\right] - \lim_{N\to 1}\frac{E(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right] - \lim_{N\to 1}\frac{r}{\gamma(N-1)^2}.$$

On evaluating the limit, we get

$$\Lambda^{-1}E\left[\frac{1}{1\cdot(\varepsilon-1)}+\frac{1}{(1-1)(1\cdot\varepsilon-(\varepsilon-1)}\right] \\ -\frac{E(\varepsilon-1)}{(1\cdot\varepsilon-(\varepsilon-1))^2}\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right]-\frac{r}{\gamma(1-1)^2},$$

which can be reduced to

$$\Lambda^{-1}E\left[\frac{1}{(\varepsilon-1)}+\frac{1}{0\cdot(\varepsilon-\varepsilon+1)}\right]-\frac{E(\varepsilon-1)}{(\varepsilon-\varepsilon+1)^2}\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right]-\frac{r}{\gamma(0)^2},$$

which further simplifies to

$$\frac{\Lambda^{-1}E}{(\varepsilon-1)}+\infty-E(\varepsilon-1)\cdot\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right]-\infty,$$

which is indeterminate. Thus, as $N\to 1$, limit of the left-hand side of Equation (89) is indeterminate.

As N aprooaches ∞, the left-hand side of Equation (89) approaches

$$\lim_{N\to\infty}\Lambda^{-1}E\left[\frac{1}{N(\varepsilon-1)}+\frac{1}{(N-1)(N\varepsilon-(\varepsilon-1)}\right] \\ -\lim_{N\to\infty}\frac{E(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right]-\lim_{N\to\infty}\frac{r}{\gamma(N-1)^2}.$$

On evaluating the limit, we obtain,

$$\Lambda^{-1}E\left[\frac{1}{\infty\cdot(\varepsilon-1)}+\frac{1}{(\infty-1)(\infty\cdot\varepsilon-\varepsilon+1)}\right] \\ -\frac{E(\varepsilon-1)}{(\infty\cdot\varepsilon-\varepsilon+1)^2}\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right]-\frac{r}{\gamma(\infty-1)^2},$$

which simplifies to

$$\Lambda^{-1}E\left[\frac{1}{\infty}+\frac{1}{\infty}\right]-\frac{E(\varepsilon-1)}{\infty}\left[1+\frac{\theta}{\gamma}\cdot(\gamma-\varepsilon)\right]-\frac{r}{\gamma\cdot\infty}=0.$$

Thus, as $N\to\infty$, limit of the left-hand side of Equation (89) is 0.

Appendix 7: Comparison of number of firms, N, across regimes

Recall Equation (78), which determines the number of firms under the free entry and exit regime,

$$\underbrace{\frac{1}{N\varepsilon-(\varepsilon-1)}}_{(a)}\underbrace{\left[1-\frac{\theta}{\gamma}\cdot\frac{(\varepsilon-1)[1+\gamma(N-1)])}{N}\right]}_{(b)}+\underbrace{\frac{\rho}{\gamma\cdot(N-1)}}_{(c)}=\phi,$$

and Equation (91), which determines the number of firms in the purely benevolent policy-maker regime,

$$\underbrace{\left[\frac{1}{N(\varepsilon-1)}+\frac{1}{(N-1)(N\varepsilon-(\varepsilon-1))}\right]}_{(d)} - \left[\underbrace{\underbrace{\frac{\varepsilon-1}{(N\varepsilon-(\varepsilon-1))^2}}_{(e)}\underbrace{\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]}_{(f)}}_{(g)}+\underbrace{\frac{\rho}{\gamma(N-1)^2}}_{(h)}\right]=\phi.$$

Also recall the parametric restrictions assumed in our model, viz., $\rho>0$, $\phi>0$, $0<\gamma<1$, $N>1$ and $\varepsilon>1$. Without loss of generality, we assume $\theta>0$, that is to say, the elasticity of quality with respect to R&D investment is positive.

We begin the analysis by comparing *Term (b)* in Equation (78) and *Term (f)* in Equation (91). Initially, let us consider

$$1-\frac{\theta}{\gamma}\cdot\frac{(\varepsilon-1)[1+\gamma(N-1)]}{N}<1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma). \tag{104}$$

Note that these expressions become equal only when $\gamma=1$, which we have eliminated by assumption. Upon subtracting 1 from both sides of Equation (104), we obtain

$$-\frac{\theta}{\gamma}\cdot\frac{(\varepsilon-1)[1+\gamma(N-1)]}{N}<-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma),$$

and on multiplying both sides by -1 we obtain

$$\frac{\theta}{\gamma}\cdot\frac{(\varepsilon-1)[1+\gamma(N-1)]}{N}>\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)$$

and on further multiplying both sides by $\frac{\gamma}{\theta}$, we arrive at

$$\frac{(\varepsilon-1)[1+\gamma(N-1)]}{N}>\varepsilon-\gamma. \tag{105}$$

Note that since $0 < \gamma < 1$ and $\theta > 0$, the inequality does not reverse on multiplying both sides by $\frac{\gamma}{\theta}$. We now multiply both sides of equation Equation (105) by N, which yields

$$(\varepsilon - 1)[1 + \gamma(N - 1)] > N(\varepsilon - \gamma). \tag{106}$$

On simplifying equation Equation (106), we obtian

$$\varepsilon + N\varepsilon\gamma - \varepsilon\gamma - 1 - N\gamma + \gamma > N\varepsilon - N\gamma,$$

which on canceling out like terms and further simplification yields

$$(\gamma - 1)\Big[\varepsilon(N - 1) + 1\Big] > 0. \tag{107}$$

Since $\varepsilon > 1$ and $N > 1$, we know that $\varepsilon(N - 1) + 1 > 0$. However, $0 < \gamma < 1$ and therefore $\gamma - 1 < 0$. Therefore, Equation (107) cannot hold true, which means that our initial consideration made in equation Equation (104) is contradicted. Therefore, the correct direction of inequality is given by

$$1 - \frac{\theta}{\gamma} \cdot \frac{(\varepsilon - 1)[1 + \gamma(N - 1)]}{N} > 1 - \frac{\theta}{\gamma} \cdot (\varepsilon - \gamma).$$

On multiplying both sides of the above equation by $\frac{1}{N\varepsilon - (\varepsilon - 1)}$ we obtain,

$$\frac{1}{N\varepsilon - (\varepsilon - 1)}\left[1 - \frac{\theta}{\gamma} \cdot \frac{(\varepsilon - 1)[1 + \gamma(N - 1)]}{N}\right] > \frac{1}{N\varepsilon - (\varepsilon - 1)}\left[1 - \frac{\theta}{\gamma} \cdot (\varepsilon - \gamma)\right]. \tag{108}$$

Note that since $N\varepsilon - (\varepsilon - 1) > 0$, the direction of inequality remains unchanged.

Consider the term

$$\frac{\varepsilon - 1}{N\varepsilon - (\varepsilon - 1)}.$$

Since $\varepsilon > 1$ and $N\varepsilon - (\varepsilon - 1) = \varepsilon(N - 1) + 1 > 0$, we have

$$\frac{\varepsilon - 1}{N\varepsilon - (\varepsilon - 1)} > 0.$$

We then check for whether $\frac{\varepsilon - 1}{N\varepsilon - (\varepsilon - 1)} \leq 1$. This would mean that

$$\varepsilon - 1 \leq N\varepsilon - (\varepsilon - 1),$$

which upon further simplification yields

$$N \geq 2\left[1 - \frac{1}{\varepsilon}\right].$$

With $\varepsilon > 1$ and $N > 1$, the above expression always holds true. Therefore, $\frac{\varepsilon-1}{N\varepsilon-(\varepsilon-1)} \leq 1$. We now multiply the right-hand side of Equation (108) with $0 < \frac{\varepsilon-1}{N\varepsilon-(\varepsilon-1)} \leq 1$ to obtain

$$\frac{1}{N\varepsilon-(\varepsilon-1)}\left[1-\frac{\theta}{\gamma}\cdot\frac{(\varepsilon-1)[1+\gamma(N-1)]}{N}\right] > \frac{\varepsilon-1}{[N\varepsilon-(\varepsilon-1)]^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]. \quad (109)$$

Next, consider the terms $\frac{\rho}{\gamma(N-1)}$ and $\frac{\rho}{\gamma(N-1)^2}$. note that

$$\frac{\rho}{\gamma(N-1)} > \frac{\rho}{\gamma(N-1)^2} \qquad \forall N > 1.$$

Therefore, when we add the left-hand side of Equation (109) with a larger number, $\frac{\rho}{\gamma(N-1)}$, and the right-hand side with a smaller number, $\frac{\rho}{\gamma(N-1)^2}$, the direction of the inequality remains the same. Therefore, we have

$$\frac{1}{N\varepsilon-(\varepsilon-1)}\left[1-\frac{\theta}{\gamma}\cdot\frac{(\varepsilon-1)[1+\gamma(N-1)]}{N}\right]+\frac{\rho}{\gamma(N-1)}$$
$$> \frac{\varepsilon-1}{[N\varepsilon-(\varepsilon-1)]^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]+\frac{\rho}{\gamma(N-1)^2}. \quad (110)$$

Note that the left-hand side of Equation (110) is the same as the free entry and exit condition given in Equation (78). Also note that the right-hand side of Equation (110) corresponds to *Term (g)* of Equation (91) (with $E = 1$ and $r = \rho$). We know from Assumption 1 that the right-hand side of Equation (110) is positive. Hence, we can re-write Equation (110) as

$$\frac{\frac{1}{N\varepsilon-(\varepsilon-1)}\left[1-\frac{\theta}{\gamma}\cdot\frac{(\varepsilon-1)[1+\gamma(N-1)]}{N}\right]+\frac{\rho}{\gamma(N-1)}}{\frac{\varepsilon-1}{[N\varepsilon-(\varepsilon-1)]^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]+\frac{\rho}{\gamma(N-1)^2}} > 1. \quad (111)$$

We now undertake a comparison of the free entry and exit and the benevolent policymaker regimes by checking for whether the number of firms as determined in the free entry and exit regime is greater than the number of firms as determined in the purely benevolent policymaker regime, that is to say, whether

$$\frac{1}{N\varepsilon-(\varepsilon-1)}\left[1-\frac{\theta}{\gamma}\cdot\frac{(\varepsilon-1)[1+\gamma(N-1)])}{N}\right]+\frac{\rho}{\gamma\cdot(N-1)} >$$
$$\left[\frac{1}{N(\varepsilon-1)}+\frac{1}{(N-1)(N\varepsilon-(\varepsilon-1))}\right]$$
$$-\left[\frac{(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]+\frac{\rho}{\gamma(N-1)^2}\right].$$

On dividing both sides of the above equation by $\left[\frac{(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]+\frac{\rho}{\gamma(N-1)^2}\right]$ (which is positive by Assumption 1), we obtain

$$\frac{\frac{1}{N\varepsilon-(\varepsilon-1)}\left[1-\frac{\theta}{\gamma}\cdot\frac{(\varepsilon-1)[1+\gamma(N-1)])}{N}\right]+\frac{\rho}{\gamma\cdot(N-1)}}{\frac{(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]+\frac{\rho}{\gamma(N-1)^2}} > \frac{\frac{1}{N(\varepsilon-1)}+\frac{1}{(N-1)(N\varepsilon-(\varepsilon-1))}}{\frac{(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]+\frac{\rho}{\gamma(N-1)^2}}-1. \quad (112)$$

Observe that the left-hand side of Equation (112) is the same as the left-hand side of Equation (111). If the right-hand side of Equation (112) is less than or equal to 1, then we know that Equation (112) always holds. We now check for the same, that is, whether

$$\frac{\frac{1}{N(\varepsilon-1)}+\frac{1}{(N-1)(N\varepsilon-(\varepsilon-1))}}{\frac{(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]+\frac{\rho}{\gamma(N-1)^2}}-1 \leq 1. \quad (113)$$

Recall from Equation (91) that

$$\left[\frac{1}{N(\varepsilon-1)}+\frac{1}{(N-1)(N\varepsilon-(\varepsilon-1))}\right] - \left[\frac{(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]+\frac{\rho}{\gamma(N-1)^2}\right]=\phi,$$

which can be re-expressed as

$$\frac{\frac{1}{N(\varepsilon-1)}+\frac{1}{(N-1)(N\varepsilon-(\varepsilon-1))}}{\left[\frac{(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]+\frac{\rho}{\gamma(N-1)^2}\right]}-1 = \frac{\phi}{\left[\frac{(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]+\frac{\rho}{\gamma(N-1)^2}\right]},$$

which can also be written as

$$\frac{\frac{1}{N(\varepsilon-1)}+\frac{1}{(N-1)(N\varepsilon-(\varepsilon-1))}}{\left[\frac{(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]+\frac{\rho}{\gamma(N-1)^2}\right]} = \frac{\phi}{\left[\frac{(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]+\frac{\rho}{\gamma(N-1)^2}\right]}+1. \quad (114)$$

On substituting Equation (114) in Equation (113), we obtain

$$\frac{\phi}{\left[\frac{(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]+\frac{\rho}{\gamma(N-1)^2}\right]}+1-1 \leq 1,$$

which simplifies to

$$\frac{\phi}{\left[\frac{(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]+\frac{\rho}{\gamma(N-1)^2}\right]} \leq 1,$$

which can also be written as

$$\phi \leq \left[\frac{(\varepsilon-1)}{(N\varepsilon-(\varepsilon-1))^2}\left[1-\frac{\theta}{\gamma}\cdot(\varepsilon-\gamma)\right]+\frac{\rho}{\gamma(N-1)^2}\right]. \tag{115}$$

Equation (115) gives us the condition when the right-hand side of Equation (112) is less than or equal to 1, which, if true, would mean that the number of firms determined under the free entry and exit regime is greater than the number of firms determined under the purely benevolent policymaker regime.

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