



# Hyperbolic geometry, Riemann surfaces and Teichmüller spaces

Global Initiative on Academic Network  
(GIAN) Course

27<sup>th</sup> January to 2<sup>nd</sup> February 2025

Jawaharlal Nehru University, India

**Overview:** Hyperbolic geometry, Riemann surfaces and the geometry of Teichmüller spaces of surfaces constitute a living subject which is at the forefront of research on geometry. This field establishes relations between different areas in mathematics including complex analysis, dynamical systems, combinatorial group theory, the study of representation of discrete groups into Lie groups, symplectic geometry and 3-dimensional topology. It also has applications in mathematical physics. A Teichmüller space carries various kinds of structures, including a Kähler metric and several Finsler metrics, and the action of a discrete group (the mapping class group of the surface).

**Course Objectives:** The goal of this course is to introduce the theory of deformations of hyperbolic surfaces and Riemann surfaces, concentrating on two metrics on Teichmüller space, the Teichmüller metric and the Thurston metric. These two metrics constitute

important current research topics in the geometry of surfaces and of Teichmüller spaces. The Thurston metric is a Finsler metric which is currently a very active subject of research and the earthquake metric has been studied very recently. We shall start with the fundamental elements of hyperbolic geometry and finish with the important results concerning these metrics. Several open questions will be presented.

**Lectures:** Tentative period: The week January 27-February 2, 2025, 14 hours of lectures by Prof. Athanase Papadopoulos (AP).

There will be two lectures by the host, Prof. Riddhi Shah (RS).

There will also be some special lectures by other experts in related areas.

Tentative Schedule: 2 to 4 hours of teaching every day (one or 2 lectures of 2 hours each), distributed over 6 days.

## **Lecture 1 (2 hours): AP**

Non-Euclidean geometry. The hyperbolic plane and the sphere. Length and area. Ideal triangles and analogues on the sphere. The question of best quasi-conformal maps and best Lipschitz maps between surfaces. Applications to problems in geography.

## **Lecture 2 (2 hours): AP**

Constructions of hyperbolic surfaces. Shearing coordinates. Parameters for hyperbolic surfaces and hyperbolic surfaces with boundary. The geometry of triangles and of right angled hexagons. Best Lipschitz maps between right angle hexagons and other hyperbolic surfaces.

## **Lectures by Host professor (2 hours): RS**

Lectures and discussions on background and related topics : topology, geometry and dynamics.

## **Lecture 3 (2 hours): AP**

The classification of surfaces of finite and infinite type. Cutting and pasting hyperbolic surfaces along closed and open geodesics. Families of hyperbolic surfaces. Fenchel-Nielsen coordinates.

## **Lecture 4 (2 hours): AP**

Teichmüller spaces of surfaces with or without boundary, of finite or of infinite type. The topology and the dimension of Teichmüller space. The length functions. Convexity properties.

## **Lecture 5 (2 hours): AP**

Measured foliations and measured laminations on surfaces. The spaces of measured foliations and the space of measured laminations on a surface. The piecewise-linear structure of the space of measured laminations. Space of geodesic laminations and of chain-recurrent geodesic laminations. The actions of mapping class groups on spaces of measured foliations and of laminations. Rigidity properties.

## **Lecture 6 (2 hours): AP**

Introduction to Finsler manifolds. The Finsler structures of the Teichmüller metric, of the Thurston metric and of the earthquake metric. The unit ball of the Thurston metric. Isometries of the Thurston metric. Comparison between these metrics.

## **Lecture 7 (2 hours): AP**

The Thurston metric on Teichmüller space. Construction of minimal stretch maps. Geodesics for Thurston's metric. Surfaces with boundary. Convexity properties. Rigidity theorems.

**Registration** The participation fee for joining the course is as follows:

JNU M.Sc. Students:	Nil
JNU Research Students (Ph.D.):	INR 1000
JNU Faculty:	INR 2000
Faculty from other Institutions:	INR 2000
Other Institutions (Research Students):	INR 2000

**Who can attend**

The course is open to advanced MSc (Math) and PhD students as well as to researchers who want to know about the field of geometry of surfaces and Teichmüller spaces

**The maximum number of seats for participation will be 100.** There will be a limited number of seats for outstation participants.

**Venue:** School of Physical Sciences, Jawaharlal Nehru University, New Delhi, India

**Registration:** [www.jnu.ac.in/gian](http://www.jnu.ac.in/gian)

The link for registration will open around **November 27, 2024.**



**Athanase Papadopoulos** is Senior Researcher (Directeur de recherche) at the Centre National de la Recherche Scientifique (France), attached at the Institut de Recherche Mathématique Avancée (Strasbourg). He is the author of more than 200 publications including 45 books. He spent two years at the Institute for Advanced Study (Princeton) and he taught at the University of Southern California, Brown University, the City University of New York, Banaras Hindu University, Tsinghua University (Beijing), the Universities of Pisa, Florence and others. His main field of interest is geometry and topology. He also published on history and philosophy of mathematics, and on mathematics and music theory.



**Riddhi Shah** is a Professor of Mathematics at the School of Physical Sciences, Jawaharlal Nehru University (JNU), New Delhi, India since 2007. She was on the faculty of School of Mathematics, Tata Institute of Fundamental Research (TIFR), Mumbai, India (1990-2007). Her research is in the areas of dynamics of group actions, probability measures on groups and the structure of groups. She has been a recipient of INSA Young Scientist Award (1995), Alexander-von-Humboldt Fellowship (1997-1998) and JSPS Fellowship for Senior Scientists (2004). She is a Fellow of the National Academy of Sciences, India (2022). She is currently the Chair of the Executive Committee of the Indian Women and Mathematics and has initiated many activities to promote teaching and research in mathematics among women.

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