Course Structure for M.Sc. in Mathematics
(Academic Year 2019 – 2020)

School of Physical Sciences,
Jawaharlal Nehru University
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1 Preamble

The School of Physical Sciences (SPS) is one of the leading departments in India in terms of research and teaching in physical sciences. The SPS faculty has made significant contributions to novel interdisciplinary areas interfacing physics, chemistry and mathematics. In addition to carrying out research in traditional areas of physics, chemistry and mathematics, the school has developed computing facilities and well-equipped research and teaching laboratories.

Six years ago, during the academic year 2010–2011, the school initiated a Pre-Ph.D./Ph.D. programme in Mathematics. The M.Sc. programme in Mathematics is going to be initiated during the academic year 2019–2020. This would be a 2 (TWO) years programme consisting of 4 (FOUR) semesters. Each course carries four credits. The courses, together with a compulsory one-semester-long project of four credits, will count to fulfill the minimum of 64 credits for the M.Sc. degree.

1.1 Minimum eligibility criteria for admission

- Candidates should have either one of the following degrees:
  1. Bachelor of Science degree in Mathematics or Bachelor of Arts degree in Mathematics under the 10 + 2 + 3/4 system with at least 55% marks or equivalent.
  2. B. Tech or B. E. (in any of the engineering branches) with at least 6.0 out of 10 CGPA or equivalent.
- For SC/ST and PWD candidates the qualifying degree is relaxed to 50% or 5.5 out of 10 CGPA or equivalent.

1.2 Selection procedure

- The eligible candidates have to appear for the JNU Entrance Examination for M.Sc in Mathematics.
- Candidates will be selected based on the JNU admission policy.
2 Programme structure

2.1 Overview

- 12 Core courses + 1 Project + 3 Electives for a total of 16 courses.
- A project is a compulsory course.
- Each course carries 4 credits for a total of 64 credits.

2.2 Semester wise course distribution

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3 Courses: core and elective

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Core Courses

Elective Courses

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Details of the core courses

4.1 Algebra I (PM401)

Prerequisites: Basic group theory, basic linear algebra

1. A quick review of Group Theory: Examples - dihedral, symmetric, permutation, quaternions and some matrix groups, such as $\text{GL}_n, \text{SL}_n$, Abelian and cyclic groups, subgroups, normal subgroups, Centralizer and normalizer of a group. Lagrange’s theorem and isomorphism theorems. Group actions, class equation, counting orbits, Cayley’s theorem, Sylow’s theorems, simplicity of alternating groups.

2. Construction of groups and classification results: Direct product, classification of finitely generated abelian groups (statement without proof), semi-direct product, classification of groups of small order (up to 15), wreath product, free groups, examples of presentations of groups.

3. Composition series, solvable groups, nilpotent groups.


5. Canonical forms: Diagonalizability and diagonalization, primary decomposition theorem, generalized eigenvectors, Jordan canonical form (statement), rational canonical form (statement).

6. Inner product spaces: Orthonormal bases, Gram-Schmidt process, linear functionals and adjoints, Hermitian, unitary and normal operators, symmetric and skew symmetric bilinear forms, groups preserving bilinear forms.

7. Additional Topics: Action of linear groups on $\mathbb{R}^n$, rigid motions, $\text{SO}(3, \mathbb{R})$ and Euler’s theorem, definition of representation of a group with examples, tensor product of vectors spaces.

Main Text Books:


Supplementary References:


4.2 Complex Analysis (PM402)

Prerequisites: Basic knowledge of real analysis

1. **Quick Review of Complex numbers:** Basic operations, conjugate, modulus, argument, exponential function, roots

2. **Holomorphic functions:** Continuity, derivative, holomorphic functions, Cauchy-Riemann differential equations, harmonic functions

3. **Elementary functions:** Polynomial and rational functions, exponential function, logarithm, trigonometric and hyperbolic functions

4. **Complex integration:** Paths and contours, integration, estimation theorem, Cauchy’s integral formula, Cauchy’s theorem, Liouville’s theorem, fundamental theorem of algebra, maximum modulus principle, Schwarz’s lemma

5. **Series:** (absolute and uniform) Convergence of series, power series, Taylor series, Laurent series, the identity principle

6. **Zeros, singularities and residues:** Classification of singularities, orders of poles and zeros, winding number, meromorphic functions, Cauchy’s residue theorem, argument principle

7. **Mappings:** Linear fractional transformations, conformal mappings

8. **Application of complex integration:** Computation of indefinite integrals

9. **Additional Topics:** Branch points, doubly periodic functions, construction of sine, cosine as an inverse of a multi-valued function, Riemann mapping theorem, Dirichlet problem, analytic continuation, multivariable complex analysis

Main Text Book:


2. J. B. Conway, Functions of one complex variable, Graduate Texts in Mathematics 159, Springer-Verlag, New York, 1995

Supplementary References:


4.3 Real Analysis (PM403)

Prerequisites: Basic knowledge of real analysis and linear algebra

1. **Quick review of basic Real Analysis:** Construction of real numbers, order on real numbers and the least upper bound property, convergence of sequence and series, power series, multiplication of series, absolute and conditional convergence, rearrangements (with proof of Riemann’s Theorem). Continuity, uniform continuity, compactness and connectedness in metric spaces. Differentiation: L’Hospital’s rule, derivatives of higher orders, Taylor’s theorem, differentiation of vector-valued functions

2. **The Riemann-Stieltjes Integral:** Definition and existence of the integral, properties of the integral, integration and differentiation, integration of vector-valued functions, rectifiable curves

3. **Sequences and Series of Functions:** Pointwise and uniform convergence, uniform convergence and continuity, uniform convergence and integration, uniform convergence and differentiation, equicontinuity, Arzela-Ascoli theorem, Stone-Weierstrass theorem

4. **Calculus of Several Variables:** Differentiation of functions of several real variables (directional derivatives, partial derivatives, differentiability and the total derivative, chain rule, Jacobian, higher derivatives, interchange of the order of differentiation, Taylor’s theorem), inverse function theorem, implicit function theorem, rank theorem, differentiation of integrals, derivatives of higher order

5. **Additional Topics:** Integration of differential forms: Integration, primitive mappings, partition of unity, change of variables, differential forms, Stokes’ theorem, closed and exact forms. Some special functions: Power series, exponential and logarithmic functions, trigonometric functions, Gamma function, Fourier series

Main text book:


Supplementary References:


5. S. Lang, Undergraduate Analysis, Second edition, Springer, 2005

4.4 Basic Topology (PM404)

Prerequisites: Basic knowledge of real analysis and metric spaces

1. Familiarity with Set Theory: Countable and uncountable sets, axiom of choice and its variants.

2. Topological Spaces and continuous functions: Topology, basis, sub-basis, Hausdorff and regular spaces, order topology, subspace topology, limit points, continuous functions, homeomorphisms, product topology and metric topology.

3. Quotient Topology: Quotient map, quotient topology, quotient space.

4. Nets: Subnets, convergence of nets

5. Connectedness and Compactness: Connectedness, path-connectedness, compactness, comparison with compactness in metric spaces via nets, local compactness and one-point compactification

6. Countability and Separation Axioms: First and second countability, separability, normality, complete regularity, Urysohn’s lemma, Tietze extension theorem, Tychonoff theorem and Stone-Cĕch compactification

7. Additional Topics: Urysohn Metrization theorem, local finiteness, Nagata-Smirnov metrization theorem, paracompactness and Smirnov metrization theorem

Main Text Books:


Supplementary References:


4.5 Algebra II (PM405)

Prerequisites: Basic knowledge of ring theory, Algebra I

1. **Review of the theory of rings:** Rings and subrings, homomorphisms, ideals, prime ideal, maximal ideal, quotient ring, examples of rings - matrix ring, division ring, polynomial rings,

2. **Further topics on rings:** Radical of an ideal, nilradical, Jacobson radical, Chinese remainder theorem, Euclidean domain, principal ideal domain, unique factorization domain, Gauss’s lemma, irreducibility criteria

3. **Modules:** Definition and examples, short exact sequences, free modules, torsion sub-modules, tensor product of modules. Structure of finitely generated modules over a PID

4. **Field theory:** Definition and examples, extension of fields, finite and infinite extensions, algebraic and transcendental extensions. Homomorphism, isomorphism, automorphism. Separable extensions, normal extensions. Splitting field of a polynomial. Extending field morphisms. Existence and uniqueness (up to isomorphism) of algebraic closure of a field. Finite fields, cyclicity of multiplicative group of a finite field

5. **Galois Theory:** Introduction to Galois Theory, examples of Galois groups, fundamental theorem of Galois theory

6. **Additional Topics:** Direct limit, inverse limit of modules. Constructions using a straight edge and a compass. Solvability of equations using the radicals

**Main Text Books:**


**Supplementary References:**

1. S. Lang, Algebra, Graduate Texts in Mathematics 211, revised third edition, Springer-Verlag, New York, 2002
2. E. Artin, Edited and supplemented with a section on applications by Arthur N. Milgram, Second edition, with additions and revisions, Fifth reprinting, Notre Dame Mathematical Lectures, No. 2, University of Notre Dame Press, South Bend, Ind. 1959

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4.6 Measure Theory (PM406)

**Prerequisites:** Real Analysis, topology

1. **Quick review of Riemann Integration**

2. **Lebesgue Measure on** $\mathbb{R}$: Outer measure, outer regularity of outer measure, Lebesgue measure, regularity of Lebesgue measure, non-measurable sets

3. **Lebesgue Integral:** Simple functions, almost everywhere property, measurable functions, integrable functions, approximation of integrable functions by step and continuous functions

4. **Convergence of sequence of functions:** Pointwise, uniform, Egorov’s theorem, Lusin’s theorem

5. **Abstract measure spaces:** Sigma algebras and measurable spaces, measures and measure spaces, completeness of a measure, measurable functions and their integration, monotone convergence theorem, Fatou’s lemma, dominated convergence theorem, modes of convergence

6. **Product measure:** product sigma algebra, sigma-finite measure spaces, existence of product measures, Tonelli’s theorem, Fubini’s theorem

7. **Additional Topics:** Lebesgue differentiation theorem, almost everywhere differentiability, absolute continuity Caratheodory’s Extension theorem for outer measures, Hahn-Kolmogorov extension theorem for pre-measures, Lebesgue-Stieltjes measure, Radon measure

**Main Text book:**

1. T. Tao, An Introduction to Measure Theory, GTM 126, American Mathematical Society, 2011

**Supplementary References:**


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4.7 Functional Analysis (PM407)

**Prerequisites:** Real Analysis, Basic Topology

1. **Hilbert spaces:** Inner product spaces, Hilbert spaces, orthogonality, Riesz representation Theorem, orthonormal sets, orthogonalization, unconditional sum, orthonormal bases, isomorphisms of Hilbert spaces, separable Hilbert spaces, direct sums of Hilbert spaces

2. **Operators on Hilbert spaces:** Examples, adjoint of an operator, invertible operators, self-adjoint operators, unitary operators, isometries, projections, compact operators

3. **Banach Spaces:** Normed spaces, equivalence of norms, some inequalities, Banach spaces, finite dimensional spaces, quotient and products of normed spaces, bounded linear operators and functionals

4. **Dual Spaces:** Hahn-Banach theorem, dual of a quotient space and a subspace, reflexive spaces

5. **Category Theorems:** Baire category theorem, open mapping theorem, closed graph theorem, principle of uniform boundedness

6. **Operators on Banach spaces:** Adjoint of an operator, annihilators, compact operators

7. **Additional Topics:** Weak topology, weak-* topology, Banach-Alaoglu theorem, Goldstine’s theorem, reflexivity in terms of weak topology, separable Banach spaces

**Main Text Books:**


**Supplementary References:**


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4.8 Discrete Mathematics (PM408)

Prerequisites: Algebra I

1. **Set theory and logic:** Basic concepts, cardinal numbers

2. **Counting:** Mathematical induction, pigeonhole principle, permutations and combinations, inclusion-exclusion principle, recurrence relations, generating functions, Polya’s theorem

3. **Graph theory:** Basic definitions, trees and distance, matchings, connectivity, graph colourings, Ramsey theory, planar graphs

4. **Cryptography:** Public key cryptography, RSA, discrete log problem

5. **Additional Topics:** Codes and encoding, error detection and correction, linear codes, cyclic codes

Main Text Book:


Supplementary References:


4.9 Probability and Statistics (PM409)

**Prerequisites:** Real Analysis, basic knowledge of combinatorics

1. **Probability and random variables:** Repeated experiments and empirical definition of probability. Sample space, events. Probability as a set function on a $\sigma$-algebra. Random variables, distribution functions and probability density functions. Expectation value, variance and higher moments. Moment generating functions, inequalities of Markov and Chebyshev.

2. **Conditional probability and independence:** Conditional probability, marginal distributions and conditional distributions. Covariance and correlation, stochastic independence.

3. **Some probability distributions:** Binomial, Poisson and normal distributions. Properties of their moments.

4. **Distributions of functions of random variables:** Sampling. Transformations of random variables, Student’s t and F distributions. Distributions of mean and variance of a sample. Expectations of functions of random variables.

5. **Limiting distributions:** Stochastic convergence of random variables. Weak and strong laws of large numbers (without proofs). Central limit theorem.

6. **Additional topics:** Hypothesis testing - Examples and definitions. Uniformly most powerful tests. Likelihood ratio tests. Statistical significance.

**Main Text Books:**


**Supplementary References:**


4.10 Computational Mathematics (PM410)

**Prerequisites:** Algebra I, real analysis, complex analysis, linear Algebra, additionally some knowledge of elementary number theory and ODE will help, but is not essential.

**Goal:** The aim is to introduce the students to algorithmic way of learning, to teach them the importance of computations and to the use of computers for implementation of a few algorithms. We will be using a few programming tools such as C++, MATLAB/SciLab, SAGE to implement a few well-known algorithms from Calculus, Number Theory, Linear Algebra, Algebra, Graph Theory, Discrete Mathematics and possibly Differential Equations and Statistics depending on the preparation and inclination of the students.

This course will have a practical component and labwork.

1. **Algorithms:** Introduction to algorithms with a few standard examples
2. **Brief Introduction to Programming:** Introduction to programming languages and computational systems such as C++, SAGE, MATLAB/SciLab
3. **Algorithms in Calculus:** Newton-Raphson iteration method for finding real root, numerical integration
4. **Algorithms in Linear Algebra:** Solving systems of linear equations, diagonalization
5. **Algorithms in Differential Equations:** Solution of ordinary differential equations, Runge-Kutta
6. **Algorithms in Number theory:** Sieve of Eratosthenes, primality tests, Euclidean algorithm, greatest common divisor, solution to Pell’s equation using continued fractions
7. **Algorithms in Graph Theory and Discrete Mathematics:** Kruskal’s algorithm, finding Eulerian cycles, sorting and searching algorithms
8. **Additional topics:** Computation of Galois groups, Gröbner bases

**Main References:**


**Supplementary References:**


4.11 Ordinary Differential Equations (PM411)

**Prerequisites:** Calculus, linear algebra, complex analysis

1. **Introduction:** Motivation, mathematical modelling. Order and degree of equations, linear and non-linear equations.


3. **(In-)Homogeneous Linear Differential Equation of order $n$:** Wronskian and its properties.


5. **Additional topics:** System of ODE - Linear homogeneous system. Fundamental matrix, exponential of the matrix. Phase space. Nonlinear system, elements of stability analysis.

**Main Text Book:**


**Supplementary References:**

1. G. F. Simmons, Differential Equations with Applications and Historical Notes, Third edition, CRC Press, 2017


4.12 Partial Differential Equations (PM412)

Prerequisites: Calculus, linear algebra, complex analysis, ordinary differential equations


Main Text Books:

1. I. Sneddon, Elements of Partial Differential Equations, Dover reprint, Dover, 2006

Supplementary References:

4.13 Project (PM413)

**Prerequisites:** Reasonably good understanding about M.Sc. first year courses; especially those related to the project topic

**Goal:** The project is of one semester duration and carries 4 credits. A student will choose a topic (either a research paper or some other advanced material related to but beyond the first year courses). The student will then learn the material under the supervision of a teacher. It is expected that the student will meet the supervisor regularly (at least once per week) and present the material that he/she has learnt and keep his/her supervisor updated with his/her progress. The student is also expected to write an expository essay of about 10 to 15 pages on the project topic and also present it to a panel of examiners at the end of the term.

**Grading scheme:** The student’s performance will be evaluated based on the presentations (working seminars to the supervisor), project essay and final presentation and the grading scheme for this course will be announced prior to the beginning of the project course.
5  Details of the elective courses

5.1  Number Theory (PM501)

Prerequisites: Algebra I, Algebra II, Real Analysis, Complex Analysis

Goal: To provide an introduction to Number Theory that is beyond an undergraduate elementary number theory course and having algebraic, algebraic geometric and also analytic components. This course will cover a few topics which will illustrate that number theory uses tools from all the various disciplines of mathematics.

1. Unique factorization and applications: $\mathbb{Z}, k[x]$, unique factorization in a principal ideal domain, study of $\mathbb{Z}[i], \mathbb{Z}[\omega]$ and $\mathbb{Z}[\frac{1+\sqrt{-5}}{2}]$

2. Congruences, structure of $(\mathbb{Z}/n\mathbb{Z})^*$

3. Quadratic reciprocity, Gauss and Jacobi Sums

4. Equations over finite fields, Hasse-Davenport relation, zeta function as a generating function of number of solutions

5. Riemann zeta function. Definition of Dirichlet $L$-functions attached to a character with the possibility of deeper study of $L$-function attached to a character of order 2

6. Diophantine equations of genus 0 over the rationals: Pythagorean triplets, Pell’s equation

7. Irrationality and transcendence of $e$ and $\pi$

8. Additional Topics: Brief introduction to algebraic number theory, a very brief introduction to elliptic curves, the group law on an elliptic curve possibly without proof of the associativity property, a few important concrete examples and computations, connection to congruence number problem and Fermat’s last theorem

Main Text Book:


Supplementary References:


5.2 Differential Topology (PM502)

Prerequisites: Linear algebra, topology, real analysis

1. Quick review of multivariable calculus: Implicit and inverse function theorems
2. Quick review of separation axioms and paracompactness: First and second countability, separability, normality, complete regularity, Urysohn lemma and paracompactness
3. Topological manifolds: Definition and some basic properties
4. Smooth manifolds: Smooth atlas, smooth structure, smooth manifolds
5. Smooth maps: Smooth maps, diffeomorphisms, bump functions, partitions of unity
6. Tangent space and the differential: Derivations, tangent space, tangent vectors to curves, differential of a smooth map
7. Vector fields: Tangent bundle, vector fields on manifolds, orientation on a manifold
8. Immersions, submersions and embeddings: Implicit and inverse function theorem for manifolds, submersions, immersions
9. Submanifolds: Embedded submanifolds and their tangent spaces, regular and critical points and values, level sets, immersed submanifolds
10. Additional Topics: Smooth manifolds with boundary and associated definitions, submanifolds of manifolds with boundary, embedded submanifolds and their tangent spaces, Lie brackets, Sard’s theorem, differential forms and integration on manifolds, Stokes’ theorem

Main Text Books:

Supplementary References:
5.3 Harmonic Analysis (PM503)

**Prerequisites:** Real analysis, complex analysis, measure theory

1. **Fourier series:** Fourier development of periodic functions, examples, Dirichlet and Fejer kernels, convergence of Fourier series, Gibb’s phenomenon, Parseval’s equation

2. **Review of measure theory and integration:** Lebesgue measure, Lebesgue integral, $L^p$ space

3. **Fourier transform:** Motivation and definition, examples, Fourier inversion formula, uniform continuity and Riemann-Lebesgue lemma, Plancherel’s theorem, Poisson summation formula, convolution theorem, differentiation of Fourier transforms, Hermite functions, Laplace transform

4. **Applications:** Filtering, differential equations, central limit theorem

5. **Additional Topics:** Spherical harmonic analysis, harmonic analysis on topological groups, Pontryagin duality

**Main Text Book:**


**Supplementary References:**


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5.4 Analytic Number Theory (PM504)

Prerequisites: Complex analysis

1. **Arithmetic functions and Dirichlet series**: The ring of arithmetic functions, Dirichlet series, important arithmetic functions, average estimates

2. **Characters**: Group characters, Dirichlet characters, detection of residue classes, Gauss sums

3. **Prime number distribution**: Infinitude of primes, Chebyshev’s bounds, Riemann zeta function, Perron’s formula, prime number theorem, Dirichlet $L$-functions, primes in arithmetic progressions

4. **Circle method**: General set up, ternary Goldbach problem, partitions

5. **Sieve methods**: Selberg’s sieve, large sieve, estimates for twin primes, estimates for twins of almost-primes

6. **Additional Topics**: Modular forms, exponential sums

Main Text Book:


Supplementary References:


5.5 Proofs (PM505)

Pre-requisites: Algebra I, real analysis, complex analysis, basic topology

Goal: To discuss the concept of a proof or what would constitute as a piece of an evidence. To introduce, explain and discuss proofs in mathematics, computer science and other disciplines, brief discussion about kinds of proofs using examples, The concept/definition of zero knowledge proof in computer science, computer assisted proofs, formal proof. Another important goal of this course will be to teach the students how to write mathematics and proofs (using examples and possibly small projects as assignments).

1. What is a proof or a piece of evidence: Proofs in different walks of life - in politics, in social sciences, in history, as per the judiciary, criminology, in biology, medical sciences, weather prediction, in physics and in mathematics - illustrated using examples in brief.

2. Proofs in mathematics:
   - Language of proofs. Introduction to logic
   - Methods of proving: Numerical, computational, by induction (various kinds), by contradiction (reductio ad absurdum), contrapositive, types of proofs - direct, indirect, constructive, non-constructive.
   - How to disprove: how to construct counterexamples?

3. Proofs in computer science: The concept of zero-knowledge proof in brief using couple of the following or similar examples (a) Using the game called “where is Waldo”? (b) Graph colouring problem and it’s solution by Wigderson et al (c) “How to explain zero knowledge protocols to your children”. The concept of a proof certificate - e.g. Pratt's certificate that certifies that a specific large number is a prime number (based on Lucas’s theorem). Size and complexity of a proof.

4. Computer Assisted Proofs: One or two examples from the below will be discussed in brief: four colour theorem, classification of finite groups, sphere packing, existence of Lorentz attractor

5. Additional Topics: Role of proofs in mathematics: Discussions around the main points of the bulletin of AMS article of W. Thurston, errors in mathematical research papers, role of speculation, conjectures, and questions in the progress of mathematics. Concept of a formal proof

Main References:

1. S. Krantz, The proof is in the pudding - The changing nature of mathematical proof, Springer-Verlag, 2011
5. T. C. Hales, Formal proof, Notices of the AMS, Vol 55, Number 11, 1370–1380 (and references therein)


7. B. Mazur, The faces of evidence (in Mathematics), Notes for the presentation and discussion at Museion, February 5, 2014

Supplementary References:


3. T. C. Hales, Jordan’s proof of the Jordan Curve Theorem, STUDIES IN LOGIC, GRAMMAR AND RHETORIC 10 (23 ), 2007


5. B. Mazur, Shadows of Evidence - An essay on science, 2013
5.6 Advanced Algebra (PM506)

Prerequisites: Algebra I, Algebra II

1. Commutative Algebra: Projective and injective modules, projective and injective resolutions, functors, chain complex, exact sequences, higher derived functors, functoriality

2. Group cohomology: Ext and Tor, group cohomology, group extensions, \( H^1, H^2 \)

3. Central simple algebras: Simple modules, Schur’s lemma, semisimple modules, central simple algebras, Wedderburn’s decomposition theorem, tensor operation

4. Brauer group: Brauer group of a field, relation of Brauer group to Galois cohomology of the field

Main Text Books:


Supplementary References:

1. N. S. Gopalkrishnan, Commutative Algebra, Oxonian Press Pvt. Ltd., New Delhi, 1984

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5.7 Algebraic Topology (PM507)

Prerequisites: Basic knowledge of topology and group theory

1. **The Fundamental group:** Homotopy of paths. Definition of the fundamental group, covering spaces, the fundamental group of a circle, retractions, Brouwer fixed point theorem for a disc

2. **Deformation Retracts and Homotopy Type**

3. **Fundamental groups of** \(n\)-spheres and some surfaces

4. **The Seifert-van Kampen Theorem:** Free products of groups, free groups, Seifert-van Kampen theorem, fundamental groups of wedge of circles and of tori

5. **Classification of Covering spaces:** Covering spaces, equivalence of covering spaces, universal covering space, covering transformations, existence of covering spaces

6. **Singular Homology:** Singular complex, singular homology groups, homotopy axiom, Hurewicz theorem relating the fundamental and homology groups

7. **Additional Topics:** Brief idea about simplicial homology, reduced homology groups, homology of spheres

Main Text Books:


Supplementary References:

5.8 Banach and Operator Algebras (PM508)

Prerequisites: Real analysis, topology and functional analysis

1. **Banach algebras**: Banach algebras, ideals and quotients, invertible elements, spectrum and spectral radius, Spectral mapping theorem, Gelfand-Mazur theorem, commutative Banach algebras and their Gelfand representations, holomorphic functional calculus, quotients, Stone-Weierstrass theorem

2. **C*-algebras**: Banach *-algebras and C*-algebras, multiplier algebra, unitization, Gelfand-Naimark representation of commutative C*-algebras, functional calculus, spectral mapping theorem, positive elements of C*-algebras

3. **Operators on Hilbert spaces**: Spectrum and other properties of normal, self adjoint, projection and unitary operators, partial isometry, polar decomposition, finite-rank and compact operators, diagonalization, Hilbert-Schmidt operators, trace-class operators

4. **The Spectral Theorem**: Spectral measures, spectral theorem for normal operators

5. **Gelfand-Naimark Representation**: Ideals in C*-algebras, approximate units, quotients, positive linear functionals, Gelfand-Naimark representation of C*-algebras

6. **Additional Topics**: von Neumann algebras - Strong and weak operator topologies, commutants, von Neumann algebras, double commutant theorem, polar decomposition, projections, Calkin algebra, pre-dual of a von Neumann algebra, Kaplansky density Theorem, Abelian von Neumann algebras

Main Text Book:


Supplementary References:


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